# Accumulated Stress Based Model for Prediction of Residual Pore Pressure

Étude et développement du modèle pour le pronostic sur l'excès de pression hydrostatique interstitielle causé par les contraintes accumulées

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ABSTRACT: Even though the important influence of pore pressure rise under cyclic loading on seismic wave propagation is recognized, effective stress analysis is rarely performed due to difficulties in selecting the parameters for the pore pressure model. In this paper, a new numerical model for predicting pore pressure under cyclic loading is developed. The advantages of the model are that it requires only the CSR – N curve in selecting its parameters and it can be can be used for any loading pattern. The accuracy of the model is validated through comprehensive comparisons with measurements.

RÉSUMÉ : L'importance de l'excès de la pression interstitielle causé par les contraintes accumulées de la propagation des ondes sismiques est bien reconnue, mais l'analyse de contrainte effective est rarement pratiquée en raison de difficultés à évaluer les paramètres pour le modèle de la pression interstitielle. Le présent article concerne le développement du nouveau modèle numérique pour le pronostic sur l'excès de pression interstitiellecausé par les contraintesdans le sol. Les avantages de ce modèle sontque nous pouvons déterminer tous les paramètres avec la courbe CSR-N et qu'il peut s'appliquer aux diverses formes de contraintes. La précision du modèle est contrôlée par comparaison avec le résultat du test.

KEYWORDS: pore water pressure, damage parameter, cyclic stress ratio, accumulated shear stress, time-domain analysis.

#### 1 INTRODUCTION

Build-up of residual excess pore water pressure in sands and silts during seismic loading causes reduction in stiffness and strength of soils and can lead to liquefaction. It may greatly influence the characteristics of ground motion propagation, stability of embankments, and seismic performance of structures such as tunnels and bridges. The importance of predicting the pore pressure has been well recognized and the characteristics of pore pressure generation for sands and silts have been extensively studied (Booker et al. 1976, Carraro et al. 2003, Derakhshandi et al. 2008, Lee and Albaisa 1974, Polito et al. 2008, Xenaki and Athanasopoulos 2003).

Various empirical models have been developed in the past to predict the generation of pore pressure under cyclic loading. The earliest models are based on the concept of cyclic stress approach, where the seismic loading is presented as uniform cyclic shear stress and the liquefaction potential is characterized by the amplitude of cyclic shear stress and number of loading cycles (Seed and Lee 1966, Seed et al. 1975b). The laboratory test that best fits the cyclic stress approach is stress controlled cyclic test. The result of a stress-controlled cyclic test is often presented in the form of CSR - N curve, where the CSRrepresents the ratio of shear stress (shear stress normalized by the effective confining pressure in a cyclic triaxial test and effective vertical stress in a simple shear test) that triggers liquefaction at the given number of cycles, N. While the stress controlled cyclic triaxial test is still the most popular method, the problems of the test procedure have been identified, which include difficulty in defining the exact state at which the liquefaction initiates, specimen non-uniformity, abrupt build-up of pore pressure at high pore pressures, different state of stresses compared to the field (Kramer 1996).

Consequent laboratory tests have shown that the controlling factor of the build-up of excess pore pressure is not cyclic shear stress, but cyclic shear strain. Strain-controlled cyclic tests, especially simple shear tests, have been increasingly used to measure the excess pore pressure under cyclic loading. Numerical models that predict pore pressure as a function of accumulated shear strain have been proposed (Dobry et al. 1985a, Dobry et al. 1985b, Finn and Bhatia 1982, Ivs ic 2006).

While the advantages of strain-controlled test procedure and strain-based pore pressure model are well recognized, it should be noted that the stress-controlled cyclic tests are still the most widely used laboratory procedure for evaluating the liquefaction potential. In the absence of the strain-controlled test measurements, the input parameters for a strain-based model cannot be determined. The difficulty in selecting the input parameters for the strain-controlled test measurements for the strain-based models is one of the reasons responsible for the seldom use of effective stress dynamic analysis in practice. In the absence of strain-controlled test data, it seems logical that an alternative pore pressure model that only requires the CSR - N curve obtained from the stress-controlled test in selecting its parameters be used.

This study proposes such a pore pressure model and presents guidelines for selecting its input parameters. A method for constructing the empirical CSR - N curve from in-situ penetration resistance in case the measured CSR - N curve is not available is also outlined. The applicability of the model is validated through comparisons with laboratory test data selected from literature and also non-published test data.

#### 2 PORE PRESSURE MODEL

One of the earliest pore pressure model, developed by Seed et al. (1975b), is defined as follows:

$$r_{u} = \frac{1}{2} + \frac{1}{\pi} \sin^{-1} \left[ 2 \left( \frac{N}{N_{L}} \right)^{1/\beta} - 1 \right]$$
(1)

where,  $r_u$  = residual pore pressure normalized to the initial effective confining stress, N = equivalent number of cycles,  $N_L$  = number of cycles required to cause liquefaction,  $\beta$  = empirical

parameter.  $N/N_L$  was termed cycle ratio. Booker et al. (1976) proposed the following simplified alternative of the equation:

$$r_{u} = \frac{2}{\pi} \arcsin\left(\frac{N}{N_{t}}\right)^{1/2\beta}$$
(2)

Since the equations are identical in shape, both will be termed Seed et al model in this paper. The Seed et al. model requires definition of three parameters, which are N,  $N_L$  and  $\beta$ . N can be determined from the ground motion time history calculated from a total stress site response analysis.  $N_L$  is most often determined from simplified liquefaction approach. Extensive tests have been performed to determine the bounds of the pore pressure measurements expressed in terms of cycle ratio and representative value of  $\beta$ . Lee and Albaisa (1974) proposed upper and lower bounds, while Booker et al. (1976) recommended  $\beta = 0.7$  for clean sands. Polito et al. (2008), based on 145 cyclic triaxial tests, proposed empirical equation for  $\beta$ .

While the shape of the Seed et al. model was shown to agree well with the measured build-up of pore pressure, the model has its limitations. The main drawback of the model is that since N and  $N_L$  have to be defined a priori, it cannot be used for a coupled numerical analysis. Another limitation of the models is that it cannot be used for non-liquefiable soils for which  $N_L$  cannot be defined.

This study proposes the following modified equation, which is based on the model of Seed et al.:

$$dr_{u} = \frac{2}{\pi} \arcsin\left[\left(\frac{dD}{D_{r_{u}=1.0}}\right)^{1/2\beta}\right]$$
(3)

where,  $dr_u =$  incremental residual pore pressure ratio, D = damage parameter,  $D_{ru=1.0}$  is the value of damage parameter D at initiation of liquefaction and  $\beta$  is an empirical constant. It should be noted that N and  $N_L$  of Eq. (2) are replaced by the damage parameters D and  $D_{ru=1.0}$ , respectively. The damage parameter, which is essentially a variable which contains parameters that define the strain / stress history and can uniquely relate to build-up of pore pressure for a given soil, is defined as follows:

$$D = \eta (SR - CSR_t)^{\alpha} \tag{4}$$

where, where SR = shear stress ratio (shear stress normalized to initial effective vertical stress),  $CSR_t$  = threshold shear stress ratio below which residual pore pressure is not generated,  $\eta =$ length of shear stress path,  $\alpha$  = calibration parameter. The equation for the damage parameter is very similar to the function proposed by Ivs'ic (2006). Two parameters for D, which are  $CSR_t$  and  $\alpha$ , should be selected from trial and error.  $CSR_t$  can be selected from visual inspection of the CSR - Ncurve. The second parameter,  $\alpha$  is calculated by averaging. The concept of the damage parameter implies that D at liquefaction for a given soil, which will be termed  $D_{ru=1.0}$  in this paper, should be a constant independent of SR. Therefore,  $D_{ru=1.0}$  of the CSR - N curve should be all identical. In reality, although the values of  $D_{ru=1,0}$  for different CSRs may be similar, but they will not be identical. In other words, it might not be possible to uniquely relate to the pore pressure using a single value of  $\alpha$  for all SRs. Through several trials, it was shown that the optimum value of  $\alpha$  can be calculated by averaging using the following equation, which is derived from Eq.(4):

$$\alpha_{\text{AD}} = \sum_{i=1}^{M-1} \left\{ \frac{\log\left(\frac{N_i}{N_{i+1}}\right)}{\left[\log\left(CSR_{i+1} - CSR_{l}\right) - \log\left(CSR_{i} - CSR_{l}\right)\right]} \right\} / M \quad (5)$$

where M = number of data points of CSR - N curve, *i* and *i*+1 denote two adjacent data points of the curve.

After the selection of the parameters, it is recommended that the corresponding CSR - N curve be back-calculated and compared to the target curve to confirm that the appropriateness of the parameters. In back-calculation, one of the measured data points is selected as the reference data point. The rest of the data points for the back-calculated CSR - N curve are calculated relative to the reference data point. The following equation can be used to calculate  $CSR_i$  for a given  $N_i$ 

$$CSR_{i} = 10^{\frac{\log\left(\frac{N_{ref}}{N_{i}}\right)}{\alpha_{Avg}} + \log\left(CSR_{ref} - CSR_{i}\right)} + CSR_{i}$$
(6)

where,  $CSR_{ref} = CSR$  of the reference data,  $N_{ref} =$  number of cycles of reference data. The full back-calculated CSR - N curve can be constructed by using Eq(4). for a range of  $N_i$ . This  $CSR_i$  and  $\alpha$  selection process should be repeated until a best fit CSR - N curve is obtained.

The applicability of the recommended procedure for selecting  $CSR_t$  and  $\alpha$  is evaluated through comparison with extensive sets of data. Figure 1 - Figure 3 compare the predicted CSR - N curves with the published data. It is shown that the recommended process provides good estimates of the measurements for all cases, which encompass a wide range of soils and densities.



Figure 1. Comparison of measured (Carraro et al. 2003) and predicted CSR - N curves: (a) Clean Ottawa sand, (b) Ottawa sand with 5% non-plastic silt, (c) Ottawa sand with 10% non-plastic silt, (d) Ottawa sand with 15% non-plastic silt.



Figure 2. Comparison of measured and predicted CSR - N curves: (a) Troncoso and Verdugo (1985), (b) Xenaki and Athanasopoulos (2003), (c) (Park et al. 1999), (d) Koester (1994).



Figure 3. Comparison of measured and predicted CSR - N curves of soil samples from Korea.



Figure 4. Comparison of measured and predicted pore pressure.

## **3** VALIDATION OF THE MODEL

The applicability of the model is evaluated through comparisons with measured data set, the CSR - N curves of which are displayed in Figure 3. The measurements and the predicted pore pressures are compared in Figure 4.  $D_{ru=1.0}$  is calculated from CSR - N curves using Eq. (5).  $\beta$  were selected by trial and error. The values of  $\beta$  are shown in listed in Figure 4. It was found that all pore pressure curves fall between the upper bound ( $\beta = 1.4$ ) and mean curve ( $\beta = 0.7$ ). No curves were shown to fall below the mean curve, consistent with observations of Polito et al. (2008). Dependence of  $\beta$  on SR was observed in three measurements (Sand 6, 7, 9), while other measurements showed no or limited influence of SR. Even in soils for which SR dependence is present, use of representative value of  $\beta$  was shown to be acceptable.

If the pore pressure model is to be implemented in a timedomain dynamic analysis program, the dependence of SR on  $\beta$ cannot be modeled since SR is not constant during a seismic loading. The variation of  $\beta$  under transient loading is not yet known. If it is expected that the soil will be largely influenced by SR, it is recommended that the effective shear strain and corresponding SR be calculated from uncoupled analysis, from which the resulting  $\beta$  is selected and applied in the model throughout the analysis.

# 4 METHOD FOR CONSTRUCTING EMPIRICAL CSR - N CURVE

The proposed pore pressure model cannot be used in the absence of measured CSR - N curve. This section describes an empirical method for constructing the CSR - N curve from insitu penetration test. This process is particularly useful since field test measurements are always available.

The penetration resistance measured from a field test, including the standard or cone penetration tests, are commonly used to determine the cyclic resisting ratio (CRR) (Robertson and Campanella 1985, Seed et al. 1983), which is defined as the minimum CSR at which the liquefaction is triggered at the given number of loading cycles. The empirical curves that relate field measured penetration resistance (e.g. blow count from standard penetration test or cone tip resistance from cone penetration test) with CRR are typically developed for a magnitude (M) = 7.5 earthquake. It is a common practice to assign a value of 15 for the equivalent number of cycles for a M = 7.5 earthquake,  $N_{M=7.5}$ , based on the recommendation of Seed et al. (1975a). Liu et al. (2001) have shown that the  $N_{M=7.5}$  ranges from 19 - 30, depending on the magnitude, epicentral distance, near fault directivity, and site effects. If the number of cycles for a M =7.5 event is determined, the field test derived CRR and  $N_{M=7}$ data set can be used as a point of the CSR - N curve. The full CSR - N curve can be constructed by assigning CRR values relative to the field test derived CRR value for a range of N values. The relative values of CRR can be calculated from the normalized CSR - N curve, which is explained in detail in the following

Liu (2001) collected CSR - N curves from a large body of literature and developed normalized CSR - N curve, where CSR was normalized by  $CSR_{N=15}$ , which represents the CSR at N = 15. The data showed that the shape of the normalized curve depends on the relative density, method of sample preparation, stress path (type of laboratory test), and compositional factors such as gradation / angularity. It was concluded that the results of simple shear tests performed at relative densities between 45 – 70%, using air/water-pluviated or moist-tamped soil samples fall within a narrow band, as shown as dotted red lines in Figure 5. Also shown are the CSR - N curves from Figure 1 and Figure 2, but normalized to  $CSR_{N=15}$ . The curves of Liu (2001) are close to the upper bound of the CSR - N curves calculated in this study up to N = 15. It is consistent with the previous findings that the cyclic triaxial tests result in flatter curve

compared to the simple shear test results. However, Liu's curves are steeper at N higher than 15. Considering that the stress path imposed by a simple shear test better represents actual soil response under vertically propagating shear waves, it is recommended that the curves of Liu be used in the design if  $N_{M=7.5}$ = 15. For other values of  $N_{M=7.5}$ , the adjusted curves proposed by Liu et al. (2001) can be used.



Figure 5. Comparison of normalized CSR - N curves shown in Figure 1 and Figure 2 and range of curves from Liu (2001).

#### 5 CONCLUSIONS

This paper presented a model for predicting the pore pressure build-up under seismic loading. The model uses the concept of damage parameter to transform the cycle ratio based pore pressure model of Seed et al., 1975, such that the model is a function of accumulated stress. The main advantage of the model is that since the damage parameter is an incremental parameter that increases with each time step, the model can be incorporated in a time domain program for performing coupled effective stress dynamic analyses subjected to transient motions. There is no need to define equivalent number of cycles a priori.

The model, which requires three parameters, is very robust since it only requires the CSR - N curve determined from stress-controlled cyclic tests. The process of selecting the parameters was also outlined in detail. The model and the parameter selection process were validated through comparisons with measurements from published and non-published laboratory test data. It was shown that the model and parameter selection process can reliably predict pore pressure generation under cyclic loading.

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