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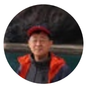

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Quadratic derivations on non-Archimedean Banach algebras

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Abstract. Let A be an algebra and X be an A-module. A quadratic mapping $D : A \to X$ is called a quadratic derivation if

$$D(ab) = D(a)b^2 + a^2 D(b)$$

for all $a_1, a_2 \in A$. We investigate the Hyers-Ulam stability of quadratic derivations from a non-Archimedean Banach algebra A into a non-Archimedean Banach A-module.

1. Introduction

A definition of stability in the case of homomorphisms between metric groups was proposed by a problem by Ulam [32] in 1940. In 1941, Hyers [17] gave a first affirmative answer to the question of Ulam for Banach spaces. Hyers' Theorem was generalized by Rassias [27] for linear mappings by considering an unbounded Cauchy difference (see [3, 4, 8, 10, 18, 19, 22, 25, 29]).

The functional equation

$$f(x + y) + f(x - y) = 2f(x) + 2f(y). \tag{1.1}$$

is related to symmetric bi-additive function. It is natural that this equation is called a quadratic functional equation. In particular, every solution of the quadratic equation (1.1) is said to be a quadratic mapping. It is well known that a mapping f between real vector spaces is quadratic if and only if there exits a unique symmetric bi-additive mapping B such that $f(x) = B(x, x)$ for all x (see [1, 20]). The bi-additive mapping B is given by

$$B(x, y) = \frac{1}{4}(f(x + y) - f(x - y)).$$

The Hyers-Ulam stability problem for the quadratic functional equation (1.1) was proved by Skof for mappings $f : A \to B$, where A is a normed space and B is a Banach space (see [31]). Cholewa [6], Czerwik [7] and Grabiec [16] have generalized the results of stability of quadratic mappings. Borelli and Forti [5] generalized the stability result as follows (cf. [23, 24]): Let G be an Abelian group, and X a Banach space. Assume that a mapping $f : G \to X$ satisfies the functional inequality

$$\|f(x + y) + f(x - y) - 2f(x) - 2f(y)\| \le \varphi(x, y)$$

for all $x, y \in G$, where $\varphi : G \times G \to [0, \infty)$ is a function such that

$$\Phi(x, y) := \sum_{i=0}^{\infty} \frac{1}{4^{i+1}} \varphi(2^i x, 2^i y) < \infty$$

for all $x, y \in G$. Then there exists a unique quadratic mapping $Q : G \to X$ with the property

$$\|f(x) - Q(x)\| \le \Phi(x, x)$$

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for all $x \in G$.

Let $\mathbb{K}$ be a field.

A non-Archimedean absolute value on $\mathbb{K}$ is a function $|\cdot| : \mathbb{K} \to \mathbb{R}$ such that for any $a, b \in \mathbb{K}$ we have

(*i*) $|a| \geq 0$ and equality holds if and only if $a = 0$,

(*ii*) $|ab| = |a||b|$,

(*iii*) $|a + b| \leq \max\{|a|, |b|\}$.

The condition (*iii*) is called the strict triangle inequality. By (*ii*), we have $|1| = |-1| = 1$. Thus, by induction, it follows from (*iii*) that $|n| \leq 1$ for each integer n. We always assume, in addition, that $|\cdot|$ is nontrivial, i.e., that there is an $a_0 \in \mathbb{K}$ such that $|a_0| \notin \{0, 1\}$.

Let X be a linear space over a scalar field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

$(NA1)$ $\|x\| = 0$ if and only if $x = 0$;

$(NA2)$ $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$;

$(NA3)$ the strong triangle inequality (ultrametric); namely,

$$\|x + y\| \leq \max\{\|x\|, \|y\|\} \qquad (x, y \in X).$$

Then $(X, \|\cdot\|)$ is called a non-Archimedean space. It follows from $(NA3)$ that

$$\|x_m - x_\ell\| \leq \max\{\|x_{\jmath+1} - x_\jmath\| : \ell \leq \jmath \leq m - 1\} \qquad (m > \ell).$$

Therefore, a sequence $\{x_m\}$ is Cauchy in X if and only if $\{x_{m+1} - x_m\}$ converges to zero in a non-Archimedean space. By a complete non-Archimedean space we mean one in which every Cauchy sequence is convergent. A non-Archimedean Banach algebra is a complete non-Archimedean algebra A which satisfies $\|ab\| \leq \|a\|\|b\|$ for all $a, b \in A$. A non-Archimedean Banach space X is a non-Archimedean Banach A-bimodule if X is an A-bimodule which satisfies $\max\{\|xa\|, \|ax\|\} \leq \|a\|\|x\|$ for all $a \in A$, $x \in X$. For more detailed definitions of non-Archimedean Banach algebras, we can refer to [30].

Let A be a normed algebra and let X be a Banach A-module. We say that a mapping $D : A \to X$ is a quadratic derivation if D is a quadratic mapping satisfying

$$D(x_1x_2) = D(x_1)x_2^2 + x_1^2D(x_2) \tag{1.2}$$

for all $x_1, x_2 \in A$.

Recently, the stability of derivations has been investigated by a number of mathematicians including [2, 11, 12, 13, 14, 15, 21, 26, 28] and references therein. More recently, Eshaghi Gordji [9] established the stability of ring derivations on non-Archimedean Banach algebras.

In this paper, we investigate the approximately quadratic derivations on non-Archimedean Banach algebras.

2. **Main results**

In the following we suppose that A is a non-Archimedean Banach algebra and X is a non-Archimedean Banach A-bimodule. Assume that $|2| \neq 1$.

Theorem 2.1. *Let $f : A \to X$ be a given mapping with $f(0) = 0$ and let $\varphi_1 : A \times A \to \mathbb{R}^+$ and $\varphi_2 : A \times A \to \mathbb{R}^+$ be functions such that*

$$\|f(x_1x_2) - f(x_1)x_2^2 - x_1^2f(x_2)\| \leq \varphi_1(x_1, x_2), \tag{2.1}$$

$$||f(x + y) + f(x - y) - 2f(x) - 2f(y)|| \leq \varphi_2(x, y) \tag{2.2}$$

for all $x_1, x_2, x, y \in A$. Assume that for each $x \in A$

$$\lim_{n\to\infty} \max\left\{\frac{1}{|2|^{2k}}\frac{\varphi_2(2^kx, 2^kx)}{|2|^2} : 0 \leq k \leq n - 1\right\}$$

denoted by $\Psi(x, x)$, exists. Suppose

$$\lim_{n\to\infty} \frac{\varphi_1(2^nx_1, 2^nx_2)}{|2|^{4n}} = \lim_{n\to\infty} \frac{\varphi_2(2^nx, 2^ny)}{|2|^{2n}} = 0$$

for all $x_1, x_2, x, y \in A$. Then there exists a unique quadratic derivation $D : A \to X$ such that

$$\|D(x) - f(x)\| \leq \Psi(x, x) \tag{2.3}$$

for all $x \in A$.

Proof. Setting $y = x$ in (2.2), we get

$$\|f(2x) - 4f(x)\| \le \varphi_2(x, x) \tag{2.4}$$

for all $x \in A$, and then dividing by $|2|^2$ in (2.4), we obtain

$$\left\|\frac{f(2x)}{2^2} - f(x)\right\| \le \frac{\varphi_2(x,x)}{|2|^2} \tag{2.5}$$

for all $x \in A$. Replacing x by $2x$ and then dividing by $|2|^2$ in (2.5), we obtain

$$\left\|\frac{f(2^2x)}{2^4} - \frac{f(2x)}{2^2}\right\| \le \frac{\varphi_2(2x,2x)}{|2|^4}. \tag{2.6}$$

Combining (2.5), (2.6) and the strong triangle inequality (NA3) yields

$$\left\|\frac{f(2^2x)}{2^4} - f(x)\right\| \le \max\left\{\frac{\varphi_2(2x,2x)}{|2|^4}, \frac{\varphi_2(x,x)}{|2|^2}\right\}. \tag{2.7}$$

Following the same argument, one can prove by induction that

$$\left\|\frac{f(2^nx)}{2^{2n}} - f(x)\right\| \le \max\left\{\frac{1}{|2|^2}\frac{\varphi_2(2^kx,2^kx)}{|2|^{2k}} : 0 \le k \le n-1\right\}. \tag{2.8}$$

Replacing x by $2^{n-1}x$ and dividing by $|2|^{2(n-1)}$ in (2.5), we find that

$$\left\|\frac{f(2^nx)}{2^{2n}} - \frac{f(2^{n-1}x)}{2^{2(n-1)}}\right\| \le \frac{\varphi_2(2^{n-1}x,2^{n-1}x)}{|2|^{2n}}$$

for all positive integers n and all $x \in A$. Hence $\{\frac{f(2^nx)}{2^{2n}}\}$ is a Cauchy sequence. Since X is complete, it follows that $\{\frac{f(2^nx)}{2^{2n}}\}$ is convergent. Set $D(x) = \lim_{n\to\infty}\frac{f(2^nx)}{2^{2n}}$. By taking the limit as $n \to \infty$ in (2.8), we see that $\|D(x) - f(x)\| \le \Psi(x,x)$ and (2.3) holds for all $x \in A$.

In order to show that D satisfies (1.2), replacing x_1, x_2 by $2^nx_1, 2^nx_2$ in (2.1), and dividing both sides of (2.1) by $|2|^{4n}$, we get

$$\left\|\frac{f(2^nx_1\cdot 2^nx_2)}{2^{4n}} - \frac{f(2^nx_1)}{2^{4n}}\cdot(2^nx_2)^2 - (2^nx_1)^2.\frac{f(2^nx_2)}{2^{4n}}\right\| \le \frac{\varphi_1(2^nx_1,2^nx_n)}{|2|^{4n}}.$$

Taking the limit as $n \to \infty$, we find that D satisfies (1.2).

Replacing x by 2^nx and y by 2^ny in (2.2) and dividing by $|2|^{2n}$, we get

$$\left\|\frac{f(2^nx+2^ny)}{2^{2n}} + \frac{f(2^nx-2^ny)}{2^{2n}} - 2\frac{f(2^nx)}{2^{2n}} - 2\frac{f(2^ny)}{2^{2n}}\right\| \le \frac{\varphi_2(2^nx,2^ny)}{|2|^{2n}}.$$

Taking the limit as $n \to \infty$, we find that D satisfies (1.1).

Now, suppose that there is another such mapping $D' : A \to X$ satisfying $D'(x+y)+D'(x-y) = 2D'(x)+2D'(y)$ and $\|D'(x) - f(x)\| \le \Psi(x,x)$. Then for all $x \in A$, we have

$$\begin{aligned}\|D(x) - D'(x)\| &= \lim_{n\to\infty}\frac{1}{|2|^{2n}}\|D(2^nx) - D'(2^nx)\| \\ &\le \lim_{n\to\infty}\frac{1}{|2|^{2n}}\max\{\|D(2^nx) - f(2^nx)\|, \|D'(2^nx) - f(2^nx)\|\} \\ &\le \lim_{n\to\infty}\lim_{k\to\infty}\frac{1}{|2|^2}\max\{\frac{\varphi_2(2^jx,2^jx)}{|2|^{2j}} : n \le j \le k+n-1\} = 0.\end{aligned}$$

It follows that $D(x) = D'(x)$. □

Corollary 2.2. *Let θ_1 and θ_2 be nonnegative real numbers, and let p be a real number such that $p > 4$. Suppose that a mapping $f : A \to X$ satisfies*

$$\|f(x_1x_2) - f(x_1)x_2^2 - x_1^2f(x_2)\| \le \theta_1(\|x_1\|^p + \|x_2\|^p),$$

$$\|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \le \theta_2(\|x\|^p + \|y\|^p)$$

for all $x_1, x_2, x, y \in A$. Then there exists a unique quadratic derivation $D : A \to X$ such that

$$\|D(x) - f(x)\| \leq \lim_{n\to\infty} \max \left\{ \frac{\theta_2 \|x\|^p}{|2|.|2|^{k(2-p)}} \; 0 \leq k \leq n-1 \right\}$$

for all $x \in A$.

Proof. Let $\varphi_1 : A \times A \to \mathbb{R}^+$ and $\varphi_2 : A \times A \to \mathbb{R}^+$ be functions such that $\varphi_1(x_1, , x_2) = \theta_1(\|x_1\|^p + \|x_2\|^p)$ and $\varphi_2(x, y) = \theta_2(\|x\|^p + \|y\|^p)$ for all $x_1, x_2, x, y \in A$. Then we have

$$\lim_{n\to\infty} \frac{\varphi_2(2^n x, 2^n y)}{|2|^{2n}} = \lim_{n\to\infty} \theta_2 \cdot |2|^{n(p-2)} \cdot (\|x\|^p + \|y\|^p) = 0 \quad (x, y \in A),$$

$$\lim_{n\to\infty} \frac{\varphi_1(2^n x_1, 2^n x_2)}{|2|^{4n}} = \lim_{n\to\infty} \frac{\theta_1 |2|^{pn}}{|2|^{4n}} (\|x_1\|^p + \|x_2\|^p) = 0 \quad (x_1, x_2 \in A).$$

Applying Theorem 2.1, we conclude the required result. □

Theorem 2.3. *Let $f : A \to X$ be a mapping and let $\varphi_1 : A \times A \to \mathbb{R}^+$, $\varphi_2 : A \times A \to \mathbb{R}^+$ be functions such that*

$$\|f(x_1 x_2) - f(x_1) x_2^2 - x_1^2 f(x_2)\| \leq \varphi_1(x_1, x_2), \tag{2.9}$$

$$\|f(x + y) + f(x - y) - 2f(x) - 2f(y)\| \leq \varphi_2(x, y) \tag{2.10}$$

for all $x_1, x_2, x, y \in A$. Assume that for each $x \in A$

$$\lim_{n\to\infty} \max \left\{ |2|^{2k} \varphi_2 \left(\frac{x}{2^{k+1}}, \frac{x}{2^{k+1}} \right) : 0 \leq k \leq n-1 \right\}$$

denoted by $\Psi(x, x)$, exists. Suppose

$$\lim_{n\to\infty} |2|^{4n} \varphi_1 \left(\frac{x_1}{2^n}, \frac{x_2}{2^n} \right) = \lim_{n\to\infty} |2|^{2n} \varphi_2 \left(\frac{x}{2^n}, \frac{y}{2^n} \right) = 0$$

for all $x_1, x_2, x, y \in A$. Then there exists a unique quadratic derivation $D : A \to X$ such that

$$\|D(x) - f(x)\| \leq \Psi(x, x) \tag{2.11}$$

for all $x \in A$.

Proof. Setting $y = x$ in (2.10), we obtain

$$\|f(2x) - 4f(x)\| \leq \varphi_2(x, x). \tag{2.12}$$

Replacing x by $\frac{x}{2}$ in (2.12), one obtains

$$\left\| f(x) - 4f\left(\frac{x}{2}\right) \right\| \leq \varphi_2 \left(\frac{x}{2}, \frac{x}{2} \right). \tag{2.13}$$

Again replacing x by $\frac{x}{2}$ in (2.13) and multiplying by $|2|^2$, we obtain that

$$\left\| 2^2 f\left(\frac{x}{2}\right) - 2^4 f\left(\frac{x}{2^2}\right) \right\| \leq |2|^2 \varphi_2 \left(\frac{x}{2^2}, \frac{x}{2^2} \right). \tag{2.14}$$

By using (2.13), (2.14) and strong triangle inequality (NA3), we get

$$\left\| f(x) - 2^4 f\left(\frac{x}{2^2}\right) \right\| \leq \max \left\{ \varphi_2 \left(\frac{x}{2}, \frac{x}{2} \right), |2|^2 \varphi_2 \left(\frac{x}{2^2}, \frac{x}{2^2} \right) \right\} \tag{2.15}$$

for $x \in A$.

Next we prove by induction that

$$\left\| f(x) - 2^{2n} f\left(\frac{x}{2^n}\right) \right\| \leq \max \left\{ |2|^{2k} \varphi_2 \left(\frac{x}{2^{k+1}}, \frac{x}{2^{k+1}} \right) : 0 \leq k \leq n-1 \right\}. \tag{2.16}$$

Replacing x by $\frac{x}{2^{n-1}}$ and multiplying by $|2|^{2(n-1)}$ in (2.13), we obtain

$$\left\| 2^{2(n-1)} f\left(\frac{x}{2^{n-1}}\right) - 2^{2n} f\left(\frac{x}{2^n}\right) \right\| \leq |2|^{2(n-1)} \varphi_2 \left(\frac{x}{2^n}, \frac{x}{2^n} \right) \tag{2.17}$$

for all $x \in A$. Hence $\{2^{2n} f(\frac{x}{2^n})\}$ is a Cauchy sequence. Since X is complete, it follows that $\{2^{2n} f(\frac{x}{2^n})\}$ is convergent. Set $D(x) = \lim_{n\longrightarrow\infty}\{2^{2n} f(\frac{x}{2^n})\}$. By taking the limit as $n \to \infty$ in (2.16), we see that $\|f(x) - D(x)\| \leq \Psi(x, x)$ and (2.11) holds for all $x \in A$.

Replacing x_1, x_2 by $\frac{x_1}{2^n}, \frac{x_2}{2^n}$ in (2.9) and multiplying by $|2|^{4n}$, we get

$$\left\|2^{4n} f\left(\frac{x_1}{2^n}\cdot\frac{x_2}{2^n}\right) - 2^{4n} f\left(\frac{x_1}{2^n}\right)\left(\frac{x_2}{2^n}\right)^2 - 2^{4n}\left(\frac{x_1}{2^n}\right)^2 f\left(\frac{x_2}{2^n}\right)\right\| \leq 2^{4n}\varphi_1\left(\frac{x_1}{2^n}, \frac{x_2}{2^n}\right).$$

Taking the limit as $n \to \infty$, we find that D satisfies (1.2).

Replacing x by $\frac{x}{2^n}$ and y by $\frac{y}{2^n}$ in (2.10) and multiplying by $|2|^{2n}$, we have

$$\left\|2^{2n} f\left(\frac{x}{2^n}+\frac{y}{2^n}\right) + 2^{2n} f\left(\frac{x}{2^n}-\frac{y}{2^n}\right) - 2^{2n}\cdot 2f\left(\frac{x}{2^n}\right) - 2^{2n}\cdot 2f\left(\frac{y}{2^n}\right)\right\| \leq |2|^{2n}\varphi_2\left(\frac{x}{2^n}, \frac{y}{2^n}\right).$$

Taking the limit as $n \to \infty$, we find that D satisfies (1.1).

Now, suppose that there is another such mapping $D' : A \to X$ satisfying $D'(x+y)+D'(x-y) = 2D'(x)+2D'(y)$ and $\|D'(x) - f(x)\| \leq \Psi(x,x)$. Then for all $x \in A$, we have

$$\begin{aligned}\|D(x) - D'(x)\| &= \lim_{n\to\infty} |2|^{2n}\left\|D\left(\frac{x}{2^n}\right) - D'\left(\frac{y}{2^n}\right)\right\| \\ &\leq \lim_{n\to\infty} |2|^{2n}\max\left\{\left\|D\left(\frac{x}{2^n}\right) - f\left(\frac{x}{2^n}\right)\right\|, \left\|D'\left(\frac{x}{2^n}\right) - f\left(\frac{x}{2^n}\right)\right\|\right\} \\ &\leq \lim_{n\to\infty}\lim_{k\to\infty}\max\left\{\varphi_2\left(\frac{x}{2^{j+1}}, \frac{x}{2^{j+1}}\right) : n \leq j \leq k+n-1\right\} = 0\end{aligned}$$

and so $D(x) = D'(x)$ for all $x \in A$. □

Corollary 2.4. *Let θ_1 and θ_2 be nonnegative real numbers, and let p be a positive real number such that $p < 2$. Suppose that a mapping $f : A \to X$ satisfies*

$$\|f(x_1x_2) - f(x_1)x_2^2 - x_1^2 f(x_2)\| \leq \theta_1(\|x_1\|^p + \|x_2\|^p),$$

$$\|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \leq \theta_2(\|x\|^p + \|y\|^p)$$

for all $x_1, x_2, x, y \in A$. Then there exists a unique quadratic derivation $D : A \to X$ such that

$$\|D(x) - f(x)\| \leq \lim_{n\to\infty}\max\{\theta_2\|x\|^p.|2|^{(k+1)(1-p)}\ 0 \leq k \leq n-1\}$$

for all $x \in A$.

Proof. Let $\varphi_1 : A \times A \to \mathbb{R}^+$ and $\varphi_2 : A \times A \to \mathbb{R}^+$ be functions such that $\varphi_1(x_1,, x_2) = \theta_1(\|x_1\|^p + \|x_2\|^p)$ and $\varphi_2(x,y) = \theta_2(\|x\|^p + \|y\|^p)$ for all $x_1, x_2, x, y \in A$. We have

$$\lim_{n\to\infty} |2|^{2n}\varphi_2\left(\frac{x}{2^n}, \frac{y}{2^n}\right) = \lim_{n\to\infty} (|2|^{n(2-p)})\theta_2(\|x\|^p + \|y\|^p) = 0 \quad (x, y \in A),$$

$$\lim_{n\to\infty} |2|^{4n}\varphi_1\left(\frac{x_1}{2^n}, \frac{x_2}{2^n}\right) = \lim_{n\to\infty} |2|^{n(4-p)}\theta_1(\|x_1\|^p + \|x_2\|^p) = 0 \quad (x_1, x_2 \in A).$$

Applying Theorem 2.4, we conclude the required result. □

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