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A functional equation related to inner product spaces in non-Archimedean $\mathcal{L}$-random normed spaces

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Abstract
In this paper, we prove the stability of a functional equation related to inner product spaces in non-Archimedean $\mathcal{L}$-random normed spaces.
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1 Introduction

One of the most interesting questions in the theory of functional analysis concerning the Ulam stability problem of functional equations is as follows: When is it true that a mapping satisfying a functional equation approximately must be close to an exact solution of the given functional equation?

The first stability problem concerning group homomorphisms was raised by Ulam [1] in 1940 and affirmatively solved by Hyers [2]. The result of Hyers was generalized by Aoki [3] for approximate additive mappings and by ThM Rassias [4] for approximate linear mappings by allowing the difference Cauchy equation $\|f(x_1+x_2)-f(x_1)-f(x_2)\|$ to be controlled by $\varepsilon(\|x_1\|^p + \|x_2\|^p)$. In 1994, a generalization of the ThM Rassias' theorem was obtained by Găvruta [5], who replaced $\varepsilon(\|x_1\|^p + \|x_2\|^p)$ by a general control function $\varphi(x_1, x_2)$.

Quadratic functional equations were used to characterize inner product spaces [6]. A square norm on an inner product space satisfies the parallelogram equality $\|x_1 + x_2\|^2 + \|x_1 - x_2\|^2 = 2(\|x_1\|^2 + \|x_1\|^2)$. The functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{1.1}$$

is related to a symmetric bi-additive mapping [7, 8]. It is natural that this equation is called a *quadratic functional equation*, and every solution of the quadratic equation (1.1) is said to be a *quadratic mapping*.

It was shown by ThM Rassias [9] that the norm defined over a real vector space X is induced by an inner product if and only if for a fixed integer $n \geq 2$

$$\sum_{i=1}^{n} \left\| x_i - \frac{1}{n}\sum_{j=1}^{n} x_j \right\|^2 = \sum_{i=1}^{n} \|x_i\|^2 - n\left\| \frac{1}{n}\sum_{i=1}^{n} x_i \right\|^2$$

for all $x_1, \ldots, x_n \in X$.

Let $\mathbb{K}$ be a field. A non-Archimedean absolute value on $\mathbb{K}$ is a function $|\cdot| : \mathbb{K} \to \mathbb{R}$ such that for any $a, b \in \mathbb{K}$ we have

(i) $|a| \geq 0$ and equality holds if and only if $a = 0$,

(ii) $|ab| = |a||b|$,

(iii) $|a + b| \leq \max\{|a|, |b|\}$.

The condition (iii) is called the strict triangle inequality. By (ii), we have $|1| = |-1| = 1$. Thus, by induction, it follows from (iii) that $|n| \leq 1$ for each integer n. We always assume in addition that $|\ |$ is non-trivial, *i.e.*, that there is an $a_0 \in \mathbb{K}$ such that $|a_0| \neq 0, 1$.

Let X be a linear space over a scalar field $\mathbb{K}$ with a non-Archimedean non-trivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

(NA1) $\|x\| = 0$ if and only if $x = 0$;

(NA2) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$;

(NA3) the strong triangle inequality (ultra-metric); namely,

$$\|x + y\| \leq \max\{\|x\|, \|y\|\} \quad (x, y \in X).$$

Then $(X, \|\cdot\|)$ is called a non-Archimedean space.

Thanks to the inequality

$$\|x_m - x_l\| \leq \max\{\|x_{\jmath+1} - x_\jmath\| : l \leq \jmath \leq m - 1\} \quad (m > l)$$

a sequence $\{x_m\}$ is Cauchy in X if and only if $\{x_{m+1} - x_m\}$ converges to zero in a non-Archimedean space. By a complete non-Archimedean space, we mean a non-Archimedean space in which every Cauchy sequence is convergent.

In 1897, Hensel [10] introduced a normed space, which does not have the Archimedean property.

During the last three decades, the theory of non-Archimedean spaces has gained the interest of physicists for their research in particular in problems coming from quantum physics, p-adic strings, and superstrings [11]. Although many results in the classical normed space theory have a non-Archimedean counterpart, but their proofs are essentially different and require an entirely new kind of intuition [12–16].

The main objective of this paper is to prove the Hyers-Ulam stability of the following functional equation related to inner product spaces:

$$\sum_{i=1}^{n} f\left(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j\right) = \sum_{i=1}^{n} f(x_i) - nf\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \tag{1.2}$$

($n \in \mathbb{N}$, $n \geq 2$) in non-Archimedean normed spaces. Interesting new results concerning functional equations related to inner product spaces have recently been obtained by Najati and ThM Rassias [18] as well as for the fuzzy stability of a functional equation related to inner product spaces by Park [19] and Gordji and Khodaei [20]. During the last decades, several stability problems for various functional equations have been investigated by many mathematicians; [21–56].

2 Preliminaries

The theory of random normed spaces (RN-spaces) is important as a generalization of the deterministic result of linear normed spaces and also in the study of random operator equations. The RN-spaces may also provide us the appropriate tools to study the geometry of nuclear physics and have important applications in quantum particle physics. The Hyers-Ulam stability of different functional equations in RN-spaces and fuzzy normed spaces has been recently studied by Alsina [57], Mirmostafaee, Mirzavaziri, and Moslehian [58, 59], Miheţ and Radu [60], Miheţ, Saadati, and Vaezpour [61, 62], Baktash *et al.* [63], Najati [64], and Saadati *et al.* [65].

Let $\mathcal{L} = (L, \geq_L)$ be a complete lattice, that is, a partially ordered set in which every nonempty subset admits supremum and infimum and $0_{\mathcal{L}} = \inf L$, $1_{\mathcal{L}} = \sup L$. The space of latticetic random distribution functions, denoted by Δ_L^+, is defined as the set of all mappings $F : \mathbb{R} \cup \{-\infty, +\infty\} \to L$ such that F is left continuous, non-decreasing on $\mathbb{R}$ and $F(0) = 0_{\mathcal{L}}$, $F(+\infty) = 1_{\mathcal{L}}$.

The subspace $D_L^+ \subseteq \Delta_L^+$ is defined as $D_L^+ = \{F \in \Delta_L^+ : l^- F(+\infty) = 1_{\mathcal{L}}\}$, where $l^- f(x)$ denotes the left limit of the function f at the point x. The space Δ_L^+ is partially ordered by the usual point-wise ordering of functions, that is, $F \geq G$ if and only if $F(t) \geq_L G(t)$ for all t in $\mathbb{R}$. The maximal element for Δ_L^+ in this order is the distribution function given by

$$\varepsilon_0(t) = \begin{cases} 0_{\mathcal{L}}, & \text{if } t \leq 0, \\ 1_{\mathcal{L}}, & \text{if } t > 0. \end{cases}$$

Definition 2.1 [66] A *triangular norm* (t-norm) on L is a mapping $\mathcal{T} : (L)^2 \longrightarrow L$ satisfying the following conditions:

(1) $(\forall x \in L)(\mathcal{T}(x, 1_{\mathcal{L}}) = x)$ (: boundary condition);
(2) $(\forall (x, y) \in (L)^2)(\mathcal{T}(x, y) = \mathcal{T}(y, x))$ (: commutativity);
(3) $(\forall (x, y, z) \in (L)^3)(\mathcal{T}(x, \mathcal{T}(y, z)) = \mathcal{T}(\mathcal{T}(x, y), z))$ (: associativity);
(4) $(\forall (x, x', y, y') \in (L)^4)(x \leq_L x'$ and $y \leq_L y' \Longrightarrow \mathcal{T}(x, y) \leq_L \mathcal{T}(x', y'))$ (: monotonicity).

Let $\{x_n\}$ be a sequence in L converging to $x \in L$ (equipped the order topology). The t-norm $\mathcal{T}$ is called a *continuous t-norm* if

$$\lim_{n\to\infty} \mathcal{T}(x_n, y) = \mathcal{T}(x, y),$$

for any $y \in L$.

A t-norm $\mathcal{T}$ can be extended (by associativity) in a unique way to an n-array operation taking for $(x_1, \dots, x_n) \in L^n$ the value $\mathcal{T}(x_1, \dots, x_n)$ defined by

$$\mathcal{T}_{i=1}^0 x_i = 1, \qquad \mathcal{T}_{i=1}^n x_i = \mathcal{T}\left(\mathcal{T}_{i=1}^{n-1} x_i, x_n\right) = \mathcal{T}(x_1, \dots, x_n).$$

The t-norm $\mathcal{T}$ can also be extended to a countable operation taking, for any sequence $\{x_n\}$ in L, the value

$$\mathcal{T}_{i=1}^\infty x_i = \lim_{n\to\infty} \mathcal{T}_{i=1}^n x_i. \tag{2.1}$$

The limit on the right side of (2.1) exists since the sequence $(\mathcal{T}_{i=1}^n x_i)_{n\in\mathbb{N}}$ is non-increasing and bounded from below.

Note that we put $\mathcal{T} = T$ whenever $L = [0,1]$. If T is a t-norm then, for all $x \in [0,1]$ and $n \in N \cup \{0\}$, $x_T^{(n)}$ is defined by 1 if $n = 0$ and $T(x_T^{(n-1)}, x)$ if $n \geq 1$. A t-norm T is said to be *of Hadžić-type* (we denote by $T \in \mathcal{H}$) if the family $(x_T^{(n)})_{n\in N}$ is equi-continuous at $x = 1$ (see [67]).

Definition 2.2 [66] A continuous t-norm $\mathcal{T}$ on $L = [0,1]^2$ is said to be *continuous t-representable* if there exist a continuous t-norm $*$ and a continuous t-co-norm $\diamond$ on $[0,1]$ such that, for all $x = (x_1, x_2)$, $y = (y_1, y_2) \in L$,

$$\mathcal{T}(x,y) = (x_1 * y_1, x_2 \diamond y_2).$$

For example,

$$\mathcal{T}(a,b) = \bigl(a_1 b_1, \min\{a_2 + b_2, 1\}\bigr)$$

and

$$\mathbf{M}(a,b) = \bigl(\min\{a_1, b_1\}, \max\{a_2, b_2\}\bigr)$$

for all $a = (a_1, a_2)$, $b = (b_1, b_2) \in [0,1]^2$ are continuous t-representable.

Define the mapping $\mathcal{T}_\wedge$ from L^2 to L by

$$\mathcal{T}_\wedge(x,y) = \min(x,y) = \begin{cases} x, & \text{if } y \geq_L x, \\ y, & \text{if } x \geq_L y. \end{cases}$$

Recall (see [67, 68]) that, if $\{x_n\}$ is a given sequence in L, then $(\mathcal{T}_\wedge)_{i=1}^n x_i$ is defined recurrently by $(\mathcal{T}_\wedge)_{i=1}^1 x_i = x_1$ and $(\mathcal{T}_\wedge)_{i=1}^n x_i = \mathcal{T}_\wedge((\mathcal{T}_\wedge)_{i=1}^{n-1} x_i, x_n)$ for all $n \geq 2$.

A *negation* on $\mathcal{L}$ is any decreasing mapping $\mathcal{N} : L \to L$ satisfying $\mathcal{N}(0_\mathcal{L}) = 1_\mathcal{L}$ and $\mathcal{N}(1_\mathcal{L}) = 0_\mathcal{L}$. If $\mathcal{N}(\mathcal{N}(x)) = x$ for all $x \in L$, then $\mathcal{N}$ is called an *involutive negation*. In the following, $\mathcal{L}$ is endowed with a (fixed) negation $\mathcal{N}$.

Definition 2.3 A *latticetic random normed space* is a triple $(X, \mu, \mathcal{T}_\wedge)$, where X is a vector space and μ is a mapping from X into D_L^+ satisfying the following conditions:

(LRN1) $\mu_x(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $x = 0$;

(LRN2) $\mu_{\alpha x}(t) = \mu_x(\frac{t}{|\alpha|})$ for all x in X, $\alpha \neq 0$ and $t \geq 0$;

(LRN3) $\mu_{x+y}(t+s) \geq_L \mathcal{T}_\wedge(\mu_x(t), \mu_y(s))$ for all $x, y \in X$ and $t, s \geq 0$.

We note that, from (LPN2), it follows that $\mu_{-x}(t) = \mu_x(t)$ for all $x \in X$ and $t \geq 0$.

Example 2.4 Let $L = [0,1] \times [0,1]$ and an operation $\leq_L$ be defined by

$$L = \bigl\{(a_1, a_2) : (a_1, a_2) \in [0,1] \times [0,1] \text{ and } a_1 + a_2 \leq 1\bigr\},$$

$$(a_1, a_2) \leq_L (b_1, b_2) \quad \Longleftrightarrow \quad a_1 \leq b_1, \qquad a_2 \geq b_2, \quad \forall a = (a_1, a_2), b = (b_1, b_2) \in L.$$

Then $(L, \leq_L)$ is a complete lattice (see [66]). In this complete lattice, we denote its units by $0_L = (0,1)$ and $1_L = (1,0)$. Let $(X, \|\cdot\|)$ be a normed space. Let $\mathcal{T}(a,b) =$

$(\min\{a_1, b_1\}, \max\{a_2, b_2\})$ for all $a = (a_1, a_2)$, $b = (b_1, b_2) \in [0,1] \times [0,1]$ and μ be a mapping defined by

$$\mu_x(t) = \left(\frac{t}{t + \|x\|}, \frac{\|x\|}{t + \|x\|}\right), \quad \forall t \in \mathbb{R}^+.$$

Then $(X, \mu, \mathcal{T})$ is a latticetic random normed space.

If $(X, \mu, \mathcal{T}_\wedge)$ is a latticetic random normed space, then we have

$$\mathcal{V} = \{V(\varepsilon, \lambda) : \varepsilon >_L 0_\mathcal{L}, \lambda \in L \setminus \{0_\mathcal{L}, 1_\mathcal{L}\}\}$$

is a complete system of neighborhoods of null vector for a linear topology on X generated by the norm F, where

$$V(\varepsilon, \lambda) = \{x \in X : F_x(\varepsilon) >_L \mathcal{N}(\lambda)\}.$$

Definition 2.5 Let $(X, \mu, \mathcal{T}_\wedge)$ be a latticetic random normed space.

(1) A sequence $\{x_n\}$ in X is said to be *convergent* to a point $x \in X$ if, for any $t > 0$ and $\varepsilon \in L \setminus \{0_\mathcal{L}\}$, there exists a positive integer N such that $\mu_{x_n - x}(t) >_L \mathcal{N}(\varepsilon)$ for all $n \geq N$.

(2) A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if, for any $t > 0$ and $\varepsilon \in L \setminus \{0_\mathcal{L}\}$, there exists a positive integer N such that $\mu_{x_n - x_m}(t) >_L \mathcal{N}(\varepsilon)$ for all $n \geq m \geq N$.

(3) A latticetic random normed space $(X, \mu, \mathcal{T}_\wedge)$ is said to be *complete* if every Cauchy sequence in X is convergent to a point in X.

Theorem 2.6 *If $(X, \mu, \mathcal{T}_\wedge)$ is a latticetic random normed space and $\{x_n\}$ is a sequence such that $x_n \to x$, then $\lim_{n\to\infty} \mu_{x_n}(t) = \mu_x(t)$.*

Proof The proof is the same as in classical random normed spaces (see [17]). □

Lemma 2.7 *Let $(X, \mu, \mathcal{T}_\wedge)$ be a latticetic random normed space and $x \in X$. If*

$$\mu_x(t) = C, \quad \forall t > 0,$$

then $C = 1_\mathcal{L}$ and $x = 0$.

Proof Let $\mu_x(t) = C$ for all $t > 0$. Since $\operatorname{Ran}(\mu) \subseteq D_L^+$, we have $C = 1_\mathcal{L}$ and, by (LRN1), we conclude that $x = 0$. □

3 Hyers-Ulam stability in non-Archimedean latticetic random spaces

In the rest of this paper, unless otherwise explicitly stated, we will assume that G is an additive group and that X is a complete non-Archimedean latticetic random space. For convenience, we use the following abbreviation for a given mapping $f : G \to X$:

$$\Delta f(x_1, \ldots, x_n) = \sum_{i=1}^{n} f\left(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j\right) - \sum_{i=1}^{n} f(x_i) + nf\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$$

for all $x_1, \ldots, x_n \in G$, where $n \geq 2$ is a fixed integer.

Lemma 3.1 [18] *Let V_1 and V_2 be real vector spaces. If an odd mapping $f : V_1 \to V_2$ satisfies the functional equation (1.2), then f is additive.*

Let $\mathcal{K}$ be a non-Archimedean field, $\mathcal{X}$ a vector space over $\mathcal{K}$ and $(\mathcal{Y}, \mu, \mathcal{T}_\wedge)$ a non-Archimedean complete *LRN*-space over $\mathcal{K}$. In the following theorem, we prove the Hyers-Ulam stability of the functional equation (1.2) in non-Archimedean latticetic random spaces for an odd mapping case.

Theorem 3.2 *Let $\mathcal{K}$ be a non-Archimedean field and $(\mathcal{X}, \mu, \mathcal{T}_\wedge)$ a non-Archimedean complete* LRN*-space over $\mathcal{K}$. Let $\varphi : G^n \to D_L^+$ be a distribution function such that*

$$\lim_{m\to\infty} \varphi_{2^m x_1, 2^m x_2, \ldots, 2^m x_n}\big(|2|^m t\big) = 1_{\mathcal{L}} = \lim_{m\to\infty} \Phi_{2^{m-1}x}\big(|2|^m t\big) \tag{3.1}$$

for all $x, x_1, x_2, \ldots, x_n \in G$, and

$$\tilde{\varphi}_x(t) = \lim_{m\to\infty} \min\big\{\Phi_{2^k x}\big(|2|^k t\big) : 0 \le k < m\big\} \tag{3.2}$$

exists for all $x \in G$, where

$$\Phi_x(t) := \min\left\{\varphi_{2x,0,\ldots,0}(t), \min\left\{\varphi_{x,x,0,\ldots,0}\left(\frac{|2|t}{n}\right), \varphi_{x,-x,\ldots,-x}\big(|2|t\big), \varphi(-x, x, \ldots, x)\right\}\right\} \tag{3.3}$$

for all $x \in G$. Suppose that an odd mapping $f : G \to X$ satisfies the inequality

$$\mu_{\Delta f(x_1,\ldots,x_n)}(t) \ge_L \varphi_{x_1,x_2,\ldots,x_n}(t) \tag{3.4}$$

for all $x_1, x_2, \ldots, x_n \in G$ and $t > 0$. Then there exists an additive mapping $A : G \to X$ such that

$$\mu_{f(x)-A(x)}(t) \ge_L \tilde{\varphi}_x\big(|2|t\big) \tag{3.5}$$

for all $x \in G$ and $t > 0$, and if

$$\lim_{\ell\to\infty} \lim_{m\to\infty} \min\big\{\Phi_{2^k x}\big(|2|^k t\big) : \ell \le k < m + \ell\big\} = 1_{\mathcal{L}} \tag{3.6}$$

then A is a unique additive mapping satisfying (3.5).

Proof Letting $x_1 = nx_1$, $x_i = nx_1'$ $(i = 2, \ldots, n)$ in (3.4) and using the oddness of f, we obtain that

$$\mu_{nf(x_1+(n-1)x_1')+f((n-1)(x_1-x_1'))-(n-1)f(x_1-x_1')-f(nx_1)-(n-1)f(nx_1')}(t) \ge_L \varphi_{nx_1,nx_1',\ldots,nx_1'}(t) \tag{3.7}$$

for all $x_1, x_1' \in G$ and $t > 0$. Interchanging x_1 with x_1' in (3.7) and using the oddness of f, we get

$$\begin{aligned} &\mu_{nf((n-1)x_1+x_1')-f((n-1)(x_1-x_1'))+(n-1)f(x_1-x_1')-(n-1)f(nx_1)-f(nx_1')}(t) \\ &\quad \ge_L \varphi_{nx_1',nx_1,\ldots,nx_1}(t) \end{aligned} \tag{3.8}$$

for all $x_1, x_1' \in G$ and $t > 0$. It follows from (3.7) and (3.8) that

$$\begin{aligned}&\mu_{nf(x_1+(n-1)x_1')-nf((n-1)x_1+x_1')+2f((n-1)(x_1-x_1'))-2(n-1)f(x_1-x_1')+(n-2)f(nx_1)-(n-2)f(nx_1')}(t)\\&\quad\geq_L \min\{\varphi_{nx_1,nx_1',\ldots,nx_1'}(t), \varphi_{nx_1',nx_1,\ldots,nx_1}(t)\}\end{aligned} \tag{3.9}$$

for all $x_1, x_1' \in G$ and $t > 0$. Setting $x_1 = nx_1$, $x_2 = -nx_1'$, $x_i = 0$ $(i = 3, \ldots, n)$ in (3.4) and using the oddness of f, we get

$$\begin{aligned}&\mu_{f((n-1)x_1+x_1')-f(x_1+(n-1)x_1')+2f(x_1-x_1')-f(nx_1)+f(nx_1')}(t)\\&\quad\geq_L \varphi_{nx_1,-nx_1',0,\ldots,0}(t)\end{aligned} \tag{3.10}$$

for all $x_1, x_1' \in G$ and $t > 0$. It follows from (3.9) and (3.10) that

$$\begin{aligned}&\mu_{f((n-1)(x_1-x_1'))+f(x_1-x_1')-f(nx_1)+f(nx_1')}(t)\\&\quad\geq_L \min\left\{\varphi_{nx_1,-nx_1',0,\ldots,0}\left(\frac{|2|}{n}\right), \varphi_{nx_1,nx_1',\ldots,nx_1'}\left(\frac{|2|}{n}\right), \varphi_{nx_1',nx_1,\ldots,nx_1}\left(\frac{|2|}{n}\right)\right\}\end{aligned} \tag{3.11}$$

for all $x_1, x_1' \in G$ and $t > 0$. Putting $x_1 = n(x_1 - x_1')$, $x_i = 0$ $(i = 2, \ldots, n)$ in (3.4), we obtain

$$\mu_{f(n(x_1-x_1'))-f((n-1)(x_1-x_1'))-f((x_1-x_1'))}(t) \geq_L \varphi_{n(x_1-x_1'),0,\ldots,0}(t) \tag{3.12}$$

for all $x_1, x_1' \in G$ and $t > 0$. It follows from (3.11) and (3.12) that

$$\begin{aligned}\mu_{f(n(x_1-x_1'))-f(nx_1)+f(nx_1')}(t) \geq_L \min\Bigg\{&\varphi_{n(x_1-x_1'),0,\ldots,0}(t), \varphi_{nx_1,-nx_1',0,\ldots,0}\left(\frac{|2|}{n}t\right),\\&\min\left\{\varphi_{nx_1,nx_1',\ldots,nx_1'}\left(\frac{|2|}{n}t\right), \varphi_{nx_1',nx_1,\ldots,nx_1}\left(\frac{|2|}{n}t\right)\right\}\Bigg\}\end{aligned} \tag{3.13}$$

for all $x_1, x_1' \in G$ and $t > 0$. Replacing x_1 and x_1' by $\frac{x}{n}$ and $\frac{-x}{n}$ in (3.13), respectively, we obtain

$$\mu_{f(2x)-2f(x)}(t) \geq_L \min\left\{\varphi_{2x,0,\ldots,0}(t), \min\left\{\varphi_{x,x,0,\ldots,0}\left(\frac{|2|}{n}t\right), \varphi_{x,-x,\ldots,-x}(t), \varphi_{-x,x,\ldots,x}(t)\right\}\right\}$$

for all $x \in G$ and $t > 0$. Hence,

$$\mu_{\frac{f(2x)}{2}-f(x)}(t) \geq_L \Phi_x(|2|t) \tag{3.14}$$

for all $x \in G$ and $t > 0$. Replacing x by $2^{m-1}x$ in (3.14), we have

$$\mu_{\frac{f(2^{m-1}x)}{2^{m-1}}-\frac{f(2^m x)}{2^m}}(t) \geq_L \Phi_{2^{m-1}x}(|2|^m t) \tag{3.15}$$

for all $x \in G$ and $t > 0$. It follows from (3.1) and (3.15) that the sequence $\{\frac{f(2^m x)}{2^m}\}$ is Cauchy. Since X is complete, we conclude that $\{\frac{f(2^m x)}{2^m}\}$ is convergent. So one can define the mapping $A : G \to X$ by $A(x) := \lim_{m\to\infty} \frac{f(2^m x)}{2^m}$ for all $x \in G$. It follows from (3.14) and (3.15) that

$$\mu_{f(x)-\frac{f(2^m x)}{2^m}}(t) \geq_L \min\{\Phi_{2^k x}(|2|^{k+1}t) : 0 \leq k < m\} \tag{3.16}$$

for all $m \in \mathbb{N}$ and all $x \in G$ and $t > 0$. By taking m to approach infinity in (3.16) and using (3.2), one gets (3.5). By (3.1) and (3.4), we obtain

$$\begin{aligned} \mu_{\Delta A(x_1,x_2,\ldots,x_n)}(t) &= \lim_{m\to\infty} \mu_{\Delta f(2^m x_1,2^m x_2,\ldots,2^m x_n)}\bigl(|2|^m t\bigr) \\ &\geq_L \lim_{m\to\infty} \varphi_{2^m x_1,2^m x_2,\ldots,2^m x_n}\bigl(|2|^m t\bigr) = 1_{\mathcal{L}} \end{aligned}$$

for all $x_1, x_2, \ldots, x_n \in G$ and $t > 0$. Thus the mapping A satisfies (1.2). By Lemma 3.1, A is additive.

If A' is another additive mapping satisfying (3.5), then

$$\begin{aligned} \mu_{A(x)-A'(x)}(t) &= \lim_{\ell\to\infty} \mu_{A(2^\ell x)-A'(2^\ell x)}\bigl(|2|^\ell t\bigr) \\ &\geq_L \lim_{\ell\to\infty} \min\bigl\{\mu_{A(2^\ell x)-f(2^\ell x)}\bigl(|2|^\ell t\bigr), \mu_{f(2^\ell x)-Q'(2^\ell x)}\bigl(|2|^\ell t\bigr)\bigr\} \\ &\geq_L \lim_{\ell\to\infty}\lim_{m\to\infty} \min\bigl\{\tilde{\varphi}_{2^k x}\bigl(|2|^{k+1}\bigr) : \ell \leq k < m+\ell\bigr\} = 0 \end{aligned}$$

for all $x \in G$, thus, $A = A'$. □

Corollary 3.3 *Let $\rho : [0,\infty) \to [0,\infty)$ be a function satisfying*

(i) $\rho(|2|t) \leq \rho(|2|)\rho(t)$ *for all* $t \geq 0$,

(ii) $\rho(|2|) < |2|$.

Let $\varepsilon > 0$ and let $(G, \mu, \mathcal{T}_\wedge)$ be an LRN*-space in which $L = D^+$. Suppose that an odd mapping $f : G \to X$ satisfies the inequality*

$$\mu_{\Delta f(x_1,\ldots,x_n)}(t) \geq_L \frac{t}{t + \varepsilon \sum_{i=1}^n \rho(\|x_i\|)}$$

for all $x_1, \ldots, x_n \in G$ and $t > 0$. Then there exists a unique additive mapping $A : G \to X$ such that

$$\mu_{f(x)-A(x)}(t) \geq_L \frac{t}{t + \frac{2n}{|2|^2}\varepsilon\rho(\|x\|)}$$

for all $x \in G$ and $t > 0$.

Proof Defining $\varphi : G^n \to D^+$ by $\varphi_{x_1,\ldots,x_n}(t) := \frac{t}{t+\varepsilon\sum_{i=1}^n \rho(\|x_i\|)}$, we have

$$\lim_{m\to\infty} \varphi_{2^m x_1,\ldots,2^m x_n}\bigl(|2|^m t\bigr) \geq_L \lim_{m\to\infty} \varphi_{x_1,\ldots,x_n}\left(\left(\frac{|2|}{\rho(|2|)}\right)^m\right) = 1_{\mathcal{L}}$$

for all $x_1, \ldots, x_n \in G$ and $t > 0$. So, we have

$$\tilde{\varphi}_x(t) := \lim_{m\to\infty} \min\bigl\{\Phi_{2^k x}\bigl(|2|^k\bigr) : 0 \leq k < m\bigr\} = \Phi_x(t)$$

and

$$\lim_{\ell\to\infty}\lim_{m\to\infty} \min\bigl\{\Phi_{2^k x}\bigl(|2|^k\bigr) : \ell \leq k < m+\ell\bigr\} = \lim_{\ell\to\infty} \Phi_{2^\ell x}\bigl(|2|^\ell\bigr) = 1_{\mathcal{L}}$$

for all $x \in G$ and $t > 0$. It follows from (3.3) that

$$\Phi_x(t) = \min\left\{\frac{t}{t + \varepsilon\rho(\|2x\|)}, \frac{t}{t + \frac{1}{|2|}2n\varepsilon\rho(\|x\|)}\right\} = \frac{|2|t}{|2|t + 2n\varepsilon\rho(\|x\|)}.$$

Applying Theorem 3.2, we conclude that

$$\mu_{f(x)-A(x)}(t) \geq_L \tilde{\varphi}_x\big(|2|t\big) = \Phi_x\big(|2|t\big) = \frac{t}{t + \frac{2n}{|2|^2}\varepsilon\rho(\|x\|)}$$

for all $x \in G$ and $t > 0$. □

Lemma 3.4 [18] *Let V_1 and V_2 be real vector spaces. If an even mapping $f : V_1 \to V_2$ satisfies the functional equation (1.2), then f is quadratic.*

In the following theorem, we prove the Hyers-Ulam stability of the functional equation (1.2) in non-Archimedean LRN-spaces for an even mapping case.

Theorem 3.5 *Let $\varphi : G^n \to D_L^+$ be a function such that*

$$\lim_{m\to\infty} \varphi_{2^m x_1, 2^m x_2, \ldots, 2^m x_n}\big(|2|^{2m}t\big) = 1_{\mathcal{L}} = \lim_{m\to\infty} \tilde{\varphi}'_{2^{m-1}x}\big(|2|^{2m}t\big) \tag{3.17}$$

for all $x, x_1, x_2, \ldots, x_n \in G$, $t > 0$ and

$$\tilde{\varphi}'_x(t) = \lim_{m\to\infty} \min\big\{\tilde{\varphi}'_{2^k x}\big(|2|^{2k}t\big) : 0 \leq k < m\big\} \tag{3.18}$$

exists for all $x \in G$ and $t > 0$ where

$$\begin{aligned}\tilde{\varphi}'_x(t) := \min\big\{&\varphi_{nx,nx,0,\ldots,0}\big(|2n-2|t\big), \varphi_{nx,0,\ldots,0}\big(|n-1|t\big),\\ &\varphi_{x,(n-1)x,0,\ldots,0}\big(|n-1|t\big), \Psi_x\big(|n-1|t\big)\big\}\end{aligned} \tag{3.19}$$

and

$$\Psi_x(t) := \min\left\{n\varphi_{nx,0,\ldots,0}\left(\frac{|2|}{n}t\right), \varphi_{nx,0,\ldots,0}\big(|2|t\big), \varphi_{0,nx,\ldots,nx}\big(|2|t\big)\right\} \tag{3.20}$$

for all $x \in G$ and $t > 0$. Suppose that an even mapping $f : G \to X$ with $f(0) = 0$ satisfies the inequality (3.4) for all $x_1, x_2, \ldots, x_n \in G$ and $t > 0$. Then there exists a quadratic mapping $Q : G \to X$ such that

$$\mu_{f(x)-Q(x)}(t) \geq_L \tilde{\varphi}'_x\big(|2|^2 t\big) \tag{3.21}$$

for all $x \in G$, $t > 0$ and if

$$\lim_{\ell\to\infty}\lim_{m\to\infty} \min\big\{\tilde{\varphi}'_{2^k x}\big(|2|^{2k}t\big) : \ell \leq k < m + \ell\big\} = 1_{\mathcal{L}} \tag{3.22}$$

then Q is a unique quadratic mapping satisfying (3.21).

Proof Replacing x_1 by nx_1, and x_i by nx_2 $(i = 2, \ldots, n)$ in (3.4) and using the evenness of f, we obtain

$$\begin{aligned} &\mu_{nf(x_1+(n-1)x_2)+f((n-1)(x_1-x_2))+(n-1)f(x_1-x_2)-f(nx_1)-(n-1)f(nx_2)}(t) \\ &\quad \geq_L \varphi_{nx_1,nx_2,\ldots,nx_2}(t) \end{aligned} \tag{3.23}$$

for all $x_1, x_2 \in G$ and $t > 0$. Interchanging x_1 with x_2 in (3.23) and using the evenness of f, we obtain

$$\begin{aligned} &\mu_{nf((n-1)x_1+x_2)+f((n-1)(x_1-x_2))+(n-1)f(x_1-x_2)-(n-1)f(nx_1)-f(nx_2)}(t) \\ &\quad \geq_L \varphi_{nx_2,nx_1,\ldots,nx_1}(t) \end{aligned} \tag{3.24}$$

for all $x_1, x_2 \in G$ and $t > 0$. It follows from (3.23) and (3.24) that

$$\begin{aligned} &\mu_{nf((n-1)x_1+x_2)+nf(x_1+(n-1)x_2)+2f((n-1)(x_1-x_2))+2(n-1)f(x_1-x_2)-nf(nx_1)-nf(nx_2)}(t) \\ &\quad \geq_L \min\bigl\{\varphi_{nx_1,nx_2,\ldots,nx_2}(t), \varphi_{nx_2,nx_1,\ldots,nx_1}(t)\bigr\} \end{aligned} \tag{3.25}$$

for all $x_1, x_2 \in G$ and $t > 0$. Setting $x_1 = nx_1$, $x_2 = -nx_2$, $x_i = 0$ $(i = 3, \ldots, n)$ in (3.4) and using the evenness of f, we obtain

$$\begin{aligned} &\mu_{f((n-1)x_1+x_2)+f(x_1+(n-1)x_2)+2(n-1)f(x_1-x_2)-f(nx_1)-f(nx_2)}(t) \\ &\quad \geq_L \varphi_{nx_1,-nx_2,0,\ldots,0}(t) \end{aligned} \tag{3.26}$$

for all $x_1, x_2 \in G$ and $t > 0$. So, it follows from (3.25) and (3.26) that

$$\begin{aligned} &\mu_{f((n-1)(x_1-x_2))-(n-1)^2f(x_1-x_2)}(t) \\ &\quad \geq_L \min\left\{\varphi_{nx_1,-nx_2,0,\ldots,0}\left(\frac{|2|}{n}\right), \varphi_{nx_1,nx_2,\ldots,nx_2}\bigl(|2|t\bigr), \varphi_{nx_2,nx_1,\ldots,nx_1}\bigl(|2|t\bigr)\right\} \end{aligned} \tag{3.27}$$

for all $x_1, x_2 \in G$ and $t > 0$. Setting $x_1 = x$, $x_2 = 0$ in (3.27), we obtain

$$\begin{aligned} &\mu_{f((n-1)x)-(n-1)^2f(x)}(t) \\ &\quad \geq_L \min\left\{\varphi_{nx,0,\ldots,0}\left(\frac{|2|}{n}t\right), \varphi_{nx,0,\ldots,0}\bigl(|2|t\bigr), \varphi_{0,nx,\ldots,nx}\bigl(|2|t\bigr)\right\} \end{aligned} \tag{3.28}$$

for all $x \in G$ and $t > 0$. Putting $x_1 = nx$, $x_i = 0$ $(i = 2, \ldots, n)$ in (3.4), one obtains

$$\mu_{f(nx)-f((n-1)x)-(2n-1)f(x)}(t) \geq_L \varphi_{nx,0,\ldots,0}(t) \tag{3.29}$$

for all $x \in G$ and $t > 0$. It follows from (3.28) and (3.29) that

$$\begin{aligned} \mu_{f(nx)-n^2f(x)}(t) \geq_L \min\biggl\{&\varphi_{nx,0,\ldots,0}(t), \varphi_{nx,0,\ldots,0}\left(\frac{|2|}{n}t\right), \\ &\varphi_{nx,0,\ldots,0}\bigl(|2|t\bigr), \varphi_{0,nx,\ldots,nx}\bigl(|2|t\bigr)\biggr\} \end{aligned} \tag{3.30}$$

for all $x \in G$ and $t > 0$. Letting $x_2 = -(n-1)x_1$ and replacing x_1 by $\frac{x}{n}$ in (3.26), we get

$$\mu_{f((n-1)x)-f((n-2)x)-(2n-3)f(x)}(t) \geq_L \varphi_{x,(n-1)x,0,\ldots,0}(t) \tag{3.31}$$

for all $x \in G$ and $t > 0$. It follows from (3.28) and (3.31) that

$$\begin{aligned}\mu_{f((n-2)x)-(n-2)^2 f(x)}(t) \geq_L \min\Bigg\{&\varphi_{x,(n-1)x,0,\ldots,0}(t), \varphi_{nx,0,\ldots,0}\left(\frac{|2|}{n}t\right),\\ &\varphi_{nx,0,\ldots,0}\big(|2|t\big), \varphi_{0,nx,\ldots,nx}\big(|2|t\big)\Bigg\}\end{aligned} \tag{3.32}$$

for all $x \in G$ and $t > 0$. It follows from (3.30) and (3.32) that

$$\mu_{f(nx)-f((n-2)x)-4(n-1)f(x)}(t) \geq_L \min\big\{\varphi_{nx,0,\ldots,0}(t), \varphi_{x,(n-1)x,0,\ldots,0}(t), \Psi_x(t)\big\} \tag{3.33}$$

for all $x \in G$ and $t > 0$. Setting $x_1 = x_2 = nx$, $x_i = 0$ $(i = 3, \ldots, n)$ in (3.4), we obtain

$$\mu_{f((n-2)x)+(n-1)f(2x)-f(nx)}(t) \geq_L \varphi_{nx,nx,0,\ldots,0}\big(|2|t\big) \tag{3.34}$$

for all $x \in G$ and $t > 0$. It follows from (3.33) and (3.34) that

$$\begin{aligned}&\mu_{f(2x)-4f(x)}(t)\\ &\quad\geq_L \min\big\{\varphi_{nx,nx,0,\ldots,0}\big(|2n-2|t\big), \varphi_{nx,0,\ldots,0}\big(|n-1|t\big),\\ &\qquad \varphi_{x,(n-1)x,0,\ldots,0}\big(|n-1|t\big), \Psi_x\big(|n-1|t\big)\big\}\end{aligned} \tag{3.35}$$

for all $x \in G$ and $t > 0$. Thus,

$$\mu_{f(x)-\frac{f(2x)}{2^2}}(t) \geq_L \tilde{\varphi}'_x\big(|2|^2 t\big) \tag{3.36}$$

for all $x \in G$ and $t > 0$. Replacing x by $2^{m-1}x$ in (3.36), we have

$$\mu_{\frac{f(2^{m-1}x)}{2^{2(m-1)}}-\frac{f(2^m x)}{2^{2m}}}(t) \geq_L \tilde{\varphi}'_{2^{m-1}x}\big(|2|^{2m} t\big) \tag{3.37}$$

for all $x \in G$ and $t > 0$. It follows from (3.17) and (3.37) that the sequence $\{\frac{f(2^m x)}{2^{2m}}\}$ is Cauchy. Since X is complete, we conclude that $\{\frac{f(2^m x)}{2^{2m}}\}$ is convergent. So, one can define the mapping $Q : G \to X$ by $Q(x) := \lim_{m\to\infty} \frac{f(2^m x)}{2^{2m}}$ for all $x \in G$. By using induction, it follows from (3.36) and (3.37) that

$$\mu_{f(x)-\frac{f(2^m x)}{2^{2m}}}(t) \geq_L \min\big\{\tilde{\varphi}'_{2^k x}\big(|2|^{2k+2} t\big) : 0 \leq k < m\big\} \tag{3.38}$$

for all $n \in \mathbb{N}$ and all $x \in G$ and $t > 0$. By taking m to approach infinity in (3.38) and using (3.18), one gets (3.21).

The rest of proof is similar to the proof of Theorem 3.2. □

Corollary 3.6 *Let $\eta : [0,\infty) \to [0,\infty)$ be a function satisfying*

(i) $\eta(|l|t) \le \eta(|l|)\eta(t)$ for all $t \ge 0$,

(ii) $\eta(|l|) < |l|^2$ for $l \in \{2, n-1, n\}$.

Let $\varepsilon > 0$ and let $(G, \mu, \mathcal{T}_\wedge)$ be a LRN*-space in which $L = D^+$. Suppose that an even mapping $f : G \to X$ with $f(0) = 0$ satisfies the inequality*

$$\mu_{\Delta f(x_1,\dots,x_n)}(t) \ge \frac{t}{t + \varepsilon \sum_{i=1}^{n} \eta(\|x_i\|)}$$

for all $x_1, \dots, x_n \in G$ and $t > 0$. Then there exists a unique quadratic mapping $Q : G \to X$ such that

$$\mu_{f(x)-Q(x)}(t) \ge \begin{cases} \frac{t}{t+\frac{2}{|2|^2}\varepsilon\eta(\|x\|)}, & \text{if } n = 2; \\ \frac{t}{t+\frac{n}{|2|^3|n-1|}\varepsilon\eta(\|nx\|)}, & \text{if } n > 2, \end{cases}$$

for all $x \in G$ and $t > 0$.

Proof Defining $\varphi : G^n \to D^+$ by $\varphi_{x_1,\dots,x_n}(t) := \frac{t}{t+\varepsilon\sum_{i=1}^{n}\eta(\|x_i\|)}$, we have

$$\lim_{m\to\infty} \varphi_{2^m x_1,\dots,2^m x_n}\big(|2|^{2m} t\big) \ge \lim_{m\to\infty} \varphi_{x_1,\dots,x_n}\left(\left(\frac{|2|^2}{\eta(|2|)}\right)^m\right) = 1_{\mathcal{L}}$$

for all $x_1, \dots, x_n \in G$ and $t > 0$. We have

$$\tilde{\varphi}'_x(t) := \lim_{m\to\infty} \min\big\{\tilde{\varphi}'_{2^k x}\big(|2|^{2k} t\big) : 0 \le k < m\big\}$$

and

$$\lim_{\ell\to\infty}\lim_{m\to\infty} \min\big\{\tilde{\varphi}'_{2^k x}\big(|2|^{2k} t\big) : \ell \le k < m + \ell\big\} = \lim_{\ell\to\infty} \tilde{\varphi}'_{2^\ell x}\big(|2|^{2\ell} t\big) = 0$$

for all $x \in G$ and $t > 0$. It follows from (3.20) that

$$\begin{aligned} \Psi_x(t) &= \min\left\{\frac{|2|t}{|2|t + 2n\varepsilon\eta(\|nx\|)}, \frac{|2|t}{|2|t + 2\varepsilon\eta(\|nx\|)}, \frac{|2|t}{|2|t + 2(n-1)\varepsilon\eta(\|nx\|)}\right\} \\ &= \frac{|2|t}{|2|t + n\varepsilon\eta(\|nx\|)}. \end{aligned}$$

Hence, by using (3.19), we obtain

$$\begin{aligned} \tilde{\varphi}'_x(t) &= \min\left\{\frac{|2n-2|t}{|2n-2|t + 2\varepsilon\eta(\|nx\|)}, \frac{|n-1|t}{|n-1|t + \varepsilon\eta(\|nx\|)}, \right. \\ &\qquad \left. \frac{|2n-2|t}{|2n-2|t + n\varepsilon\eta(\|nx\|)}, \frac{|n-1|t}{|n-1|t + \varepsilon(\eta(\|x\|) + \eta(\|(n-1)x\|))}\right\} \\ &= \begin{cases} \frac{t}{t+2\varepsilon\eta(\|x\|)}, & \text{if } n = 2; \\ \frac{|2||n-1|t}{|2||n-1|t+n\varepsilon\eta(\|nx\|)}, & \text{if } n > 2, \end{cases} \end{aligned}$$

for all $x \in G$ and $t > 0$. Applying Theorem 3.5, we conclude the required result. □

Lemma 3.7 [18] *Let V_1 and V_2 be real vector spaces. A mapping $f : V_1 \to V_2$ satisfies (1.2) if and only if there exist a symmetric bi-additive mapping $B : V_1 \times V_1 \to V_2$ and an additive mapping $A : V_1 \to V_2$ such that $f(x) = B(x,x) + A(x)$ for all $x \in V_1$.*

Now, we are ready to prove the main theorem concerning the Hyers-Ulam stability problem for the functional equation (1.2) in non-Archimedean spaces.

Theorem 3.8 *Let $\varphi : G^n \to D_L^+$ be a function satisfying (3.1) for all $x, x_1, x_2, \dots, x_n \in G$, and $\tilde{\varphi}_x(t)$ and $\tilde{\varphi'}_x(t)$ exist for all $x \in G$ and $t > 0$, where $\tilde{\varphi}_x(t)$ and $\tilde{\varphi'}_x(t)$ are defined as in Theorems 3.2 and 3.5. Suppose that a mapping $f : G \to X$ with $f(0) = 0$ satisfies the inequality (3.4) for all $x_1, x_2, \dots, x_n \in G$. Then there exist an additive mapping $A : G \to X$ and a quadratic mapping $Q : G \to X$ such that*

$$\mu_{f(x)-A(x)-Q(x)}(t) \geq_L \min\left\{\tilde{\varphi}_x\left(|2|^2 t\right), \tilde{\varphi}_{-x}\left(|2|^2 t\right), \tilde{\varphi'}_x\left(|2|t\right), \frac{1}{|2|}\tilde{\varphi'}_{-x}\left(|2|t\right)\right\} \tag{3.39}$$

for all $x \in G$ and $t > 0$. If

$$\begin{aligned}&\lim_{\ell\to\infty}\lim_{m\to\infty}\min\left\{\varphi_{2^k x}\left(|2|^k t\right) : \ell \leq k < m + \ell\right\}\\ &\quad = 1_{\mathcal{L}} = \lim_{\ell\to\infty}\lim_{m\to\infty}\min\left\{\tilde{\varphi'}_{2^k x}\left(|2|^{2k} t\right) : \ell \leq k < m + \ell\right\}\end{aligned}$$

then A is a unique additive mapping and Q is a unique quadratic mapping satisfying (3.39).

Proof Let $f_e(x) = \frac{1}{2}(f(x) + f(-x))$ for all $x \in G$. Then

$$\begin{aligned}\left\|\Delta f_e(x_1, \dots, x_n)\right\| &= \left\|\frac{1}{2}\left(\Delta f(x_1, \dots, x_n) + \Delta f(-x_1, \dots, -x_n)\right)\right\|\\ &\leq \frac{1}{|2|}\max\left\{\varphi(x_1, \dots, x_n), \varphi(-x_1, \dots, -x_n)\right\}\end{aligned}$$

for all $x_1, x_2, \dots, x_n \in G$ and $t > 0$. By Theorem 3.5, there exists a quadratic mapping $Q : G \to X$ such that

$$\mu_{f_e(x)-Q(x)}(t) \geq_L \min\left\{\tilde{\varphi'}_x\left(|2|^3 t\right), \tilde{\varphi'}_{-x}\left(|2|^3 t\right)\right\} \tag{3.40}$$

for all $x \in G$ and $t > 0$. Also, let $f_o(x) = \frac{1}{2}(f(x) - f(-x))$ for all $x \in G$. By Theorem 3.2, there exists an additive mapping $A : G \to X$ such that

$$\mu_{f_o(x)-A(x)}(t) \geq_L \min\left\{\tilde{\varphi}_x\left(|2|^2 t\right), \tilde{\varphi}_{-x}\left(|2|^2 t\right)\right\} \tag{3.41}$$

for all $x \in G$ and $t > 0$. Hence (3.39) follows from (3.40) and (3.41).

The rest of proof is trivial. □

Corollary 3.9 *Let $\gamma : [0, \infty) \to [0, \infty)$ be a function satisfying*

(i) $\gamma(|l|t) \leq \gamma(|l|)\gamma(t)$ for all $t \geq 0$,

(ii) $\gamma(|l|) < |l|^2$ for $l \in \{2, n-1, n\}$.

Let $\varepsilon > 0$, $(G, \mu, \mathcal{T}_\wedge)$ *be an* LRN-*space in which* $L = D^+$ *and let* $f : G \to X$ *satisfy*

$$\mu_{\Delta f(x_1,\ldots,x_n)}(t) \geq \frac{t}{t + \varepsilon \sum_{i=1}^{n} \gamma(\|x_i\|)}$$

for all $x_1, \ldots, x_n \in G$, $t > 0$ *and* $f(0) = 0$. *Then there exist a unique additive mapping* $A : G \to X$ *and a unique quadratic mapping* $Q : G \to X$ *such that*

$$\mu_{f(x)-A(x)-Q(x)}(t) \geq \frac{|2|^3 t}{|2|^3 t + 2n\varepsilon\gamma(\|x\|)}$$

for all $x \in G$ *and* $t > 0$.

Proof The result follows from Corollaries 3.6 and 3.3. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors carried out the proof. All authors conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

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