# Informational role of investment and liquidation values

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#### Abstract

We develop a credit-risk model to study the informational role of investment in an economy susceptible to large liquidity shocks. Firms' investment decisions carry information about their asset quality, thereby mitigating informational frictions when firms enter bankruptcy. An increase in aggregate investment can reduce the informational value of investment, depressing firms' recovery values. Therefore, policies boosting investment can decrease debt and firm values by reducing the informational value of investment. The presence of debt overhang may enhance firm value by making firms' investment decisions more informative. We present suggestive empirical evidence consistent with model predictions on the relation between firms' investments and recovery rates.

**Keywords** Debt overhang, Investment, Information asymmetry, Liquidation values **Paper type** Research paper

# 1. Introduction

Informational frictions can arise in a secondary market for the assets of distressed firms because potential buyers have limited time and accessibility to inside information to evaluate those assets accurately [1]. The information asymmetry between buyers and sellers can lead to inefficient asset allocation (Leland and Pyle, 1977) or even market breakdown (Akerlof, 1970), resulting in lower recovery values in bankruptcy. The informational problem is more severe during economic recessions — such as the recent Covid-19 crisis — because both *ex ante* well-managed and poorly-managed firms are financially distressed, making it more challenging to distinguish high-quality firms from low-quality firms in bankruptcy proceedings. While corporate credit ratings can provide useful information to investors, credit ratings alone are insufficient because they are primarily based on noisy financial statements and are coarse; information asymmetry exists even among firms with the same credit rating. Therefore, potential buyers in the secondary asset market are motivated to find other means to improve the assessment of failed assets.

In this paper, we propose an informational channel through which a firm's past investment decision indirectly affects the liquidation value of the firm assets in default by revealing some information about the firm's current fundamental value. In our theory,

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Journal of Derivatives and Quantitative Studies: 선물연구 Emerald Publishing Limited e-ISSN: 2713-6647 p-ISSN: 1229-988X DOI 10.1108/JDQS-02-2024-0007 because firms with different asset quality have different incentives to make investments, one can draw inferences about a firm's asset quality from its investment decisions. The information in the firm's past investment decisions can be valuable when it attempts to sell its assets in a secondary asset market with information asymmetry. Through the informational channel, a firm's prior-to-default investment decision can affect its recovery value in default and its *ex ante* debt and firm values.

To study the informational effect of investment, we develop a continuous-time credit-risk model with information asymmetry in the secondary asset market. The firm has existing assets that generate cash flow, which we interpret as the firm's asset quality and is observed only by the firm's manager and not outsiders. The firm is financed by both equity and debt, whereby the latter is a fixed amount of perpetual debt that demands a fixed amount of coupon payment. The firm has an investment opportunity at every instance in time, which may be undertaken with some fixed cost. After paying the cost, the investment boosts the growth rate of the firm's cash flows. The investment decision is controlled by the firm's manager, who acts in the best interest of the firm's equityholders.

Most importantly, we assume the firm's investment decision, whether or not the firm made an investment, is publicly observable. An interpretation of the investment in the model is an investment in physical capital or an acquisition of intangible assets. Alternatively, we can interpret it more broadly as any operational efforts that benefit the firm's long-term growth, such as expenses in advertisement and marketing, recruitment efforts in the labor market, and reforms of organizational structures and operating processes.

Firms with different cash flow levels have different incentives for making the investment. Due to the fixed-cost component in the investment costs, the firm's manager chooses not to invest when the fundamental is below an endogenously determined investment threshold. As firms with different cash flow levels make differentiated investment decisions, a firm's investment decision carries information about the firm's fundamentals. This assumption is reasonable for various interpretations of investment. For instance, firms' investments in tangible assets can be easily verified through the physical presence of new factories, facilities, and equipment. Besides, firms' costly efforts in advertisement or employee recruitment are also observable due to the nature of these investments.

The firm defaults for two distinct reasons. First, due to limited liability, the firm endogenously defaults on debt payment when the equity value falls below zero. Second, when the firm's fundamental deteriorates, we assume it occasionally fails to obtain external funding to cover operating losses. We view this as a change in financial market conditions exogenous to the firm's quality. This financial friction exposes all firms running operating losses to the risk of liquidity-driven default. Unlike in a world with only endogenous defaults, where outsiders can perfectly infer the firm's quality at default, the presence of failed firms due to this exogenous liquidity-driven default obfuscates the potential buyers' ability to infer a failed firm's quality perfectly. Instead, they can only determine a range of possible asset qualities.

When the firm defaults, its equityholders are wiped out, and a representative debtholder takes over its assets. However, the debtholder lacks the skill to manage the assets, so she attempts to sell the firm's assets to potential buyers who can operate those assets in a more efficient manner. We assume that the debtholder is perfectly informed about the firm's asset quality and that potential asset buyers are not. The information asymmetry can be interpreted as the debtholder inheriting private information from the pre-default firm manager. Alternatively, it can also be interpreted as the current management handling the bankruptcy process on behalf of the debtholder in control [2]. Under these interpretations, a gap in information between the debtholder and potential buyers arises. In addition, the debtholder cannot credibly communicate the quality of assets to the potential buyers. Instead, to overcome this information friction, the debtholder chooses the fraction of ownership to retain in a sale as a credible signal of asset quality when selling to potential buyers.

The presence of information asymmetry in the secondary asset market makes the information contained in the firm's investment decisions valuable. Although potential buyers are uninformed about a defaulted firm's asset quality, they can observe the firm's prior-to-default investment decision and rationally infer whether the firm's asset quality is above or below the investment threshold. As a result, the debtholders of firms that have made investments before default do not need to retain as large a fraction of assets to differentiate themselves from the debtholders of firms that have not made investments before default. This informational role of investment, however, also produces a negative spillover effect. When more relatively low-quality firms choose to invest, the debtholders of relatively high-quality firms can no longer be distinguished from low-quality firms by their investment decisions, so they must retain larger fractions of their assets to signal their asset quality. Consequently, more investments from low-quality firms result in lower recovery values for high-quality firms.

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Our theory generates a novel informational spillover effect on bankruptcy recovery rates across firms with different asset qualities, for which we show suggestive empirical evidence. Theoretically, when low-quality firms find it more profitable to invest, the informational value of investment diminishes because high-quality firms can no longer be distinguished by their making the investment in the secondary market. Therefore, more investments from low-quality firms have a negative spillover effect on the recovery values of high-quality firms. Empirically, we find that a higher average *ex ante* investment rate – measured as the median among firms with a specific credit rating – of B-rated firms predicts a lower average recovery value of Ba-rated firms. This relation persists even when controlling for own-credit-rating *ex ante* investment rates. This empirical finding is consistent with the informational spillover effect in our model. According to our model, if the *ex ante* credit ratings (Ba and B) do not provide perfect information on a firm's quality, a higher investment rate by B-rated firms can reduce the information value of *ex ante* investment made by Ba-rated firms, thereby lowering Ba-rated firms' recovery values in default.

This informational spillover generates three implications for government and corporate policy. First, a decrease in investment cost can reduce firm and debt value. While reducing investment costs lowers the default risk of a firm by inducing more positive-net present value (NPV) investments, such a policy may decrease the firm's recovery value in default by weakening the informational value of the firm's investment decisions. To study the model's quantitative implication, we calibrate the model to focus on firms whose bonds are in the high-yield category. We find that a decrease in investment cost induces more firms with credit ratings below Ba to invest, thereby hurting the recovery values of Ba-rated firms. Under certain situations, the adverse effect can be so large that such a cost decrease can lower the total values of Ba-rated firms. In practice, our model suggests a government policy designed to induce more corporate investments can depress the values of relatively high-quality firms. To avoid this perverse outcome, policies that encourage more investments should not provide excessive investment incentives to firms facing substantial default risks.

Second, a firm's management may not be able to increase firm value by making investments following the simple NPV rule where firms invest if the NPV of a project is positive and not otherwise. A standard model suggests a simple NPV rule of investment can increase firm value by inducing more positive NPV investments. However, according to our theory, because investment also indirectly contains an informational value in addition to its pure effects on cash flows, more investments under the simple NPV rule can make a firm's investment decision less informative and decrease its recovery value in default.

Third, our model sheds new light on the positive role of debt overhang. In the classic debt overhang problem in Myers (1977), if a firm's investment decisions are made to maximize equity value, the firm may forgo positive-NPV investments because of the conflict of interest between equity and debt. However, the spillover effect in our model implies that the underinvestment problem induced by debt overhang can mitigate information asymmetry in the

secondary asset market. Moreover, the presence of debt-equity conflict can be valueenhancing when the efficiency gain from the reduced information frictions of debt overhang outweighs the efficiency loss caused by the under-investment problem of debt overhang.

# 1.1 Literature Review

This paper is closely related to a broad literature on the effect of information asymmetry in finance. Leland and Pyle (1977) highlight inefficient asset retention as an important cost of information asymmetry in asset trading. Subsequent research finds that this asset retention problem can be mitigated by designing less informationally sensitive securities (DeMarzo and Duffie, 1999) or cross-signaling when trading multiple assets with correlated returns (He, 2009). Our paper contributes to the literature by studying the informational role of firms' investments in reducing inefficient asset retention caused by information asymmetry.

The informational role of investment in our model differs from the signaling role of investment, which is well documented in the literature. The signaling role of investment emphasizes the firms' ability to intentionally signal asset quality through their investment financing choices, investment decisions, or their timings of exercising investment options (Myers, 1984; Morellec and Schürhoff, 2011; Grenadier and Malenko, 2011). In contrast, the informational role of investment in our paper highlights how a firm's investment decision unintentionally reveals its fundamental value and thereby affects the degree of information asymmetry in the secondary asset market.

Our paper focuses on the informational value of investment in the bankruptcy process of distressed firms. In the literature, a few papers have studied the information problem when firms face financial distress [3]. Giammarino (1989) finds that the presence of asymmetric information can cause a distressed firm to forgo an informal debt restructuring and enter into a formal reorganization process that is more costly. Brown *et al.* (1993) show the information problem in distressed firms' debt restructuring can be mitigated by firms signaling their types through the choice of securities in restructuring. Nishihara and Shibata (2018) consider firms using the timing of bankruptcy to signal asset quality to outsiders. Our paper contributes to that literature by stressing the role of firms' prior-to-default investment decisions in mitigating information problems in bankruptcy.

This paper also contributes to the literature on debt overhang and corporate investment. Following the pioneering work of Myers (1977) on the under-investment problem caused by debt overhang, many papers explore and quantify the negative effects of debt overhang [4]. Hennessy (2004) shows that debt overhang distorts both the level of investment and that its composition as the under-investment problem is more severe for longer-lived assets. Hennessy et al. (2007) derive measures of costs of debt overhang from a Q theoretic model of investment and find a negative correlation between firms' investment and their debt overhang costs in the data. Chen and Manso (2017) show the cost of debt overhang is higher in the presence of macroeconomic risks and estimate the cost of debt overhang over business cycles. Most of these papers find debt overhang causes significant amounts of efficiency losses. Our paper contributes to the literature by introducing a positive role of debt overhang, which means the under-investment problem caused by debt overhang makes firms' investment decisions more informative. Our paper also shows that government subsidies or tax cuts may adversely affect creditors due to this informational role of investment. See also Doh (2023) who show different channels through which government subsidies may undesirably harm creditors and lower firm value in the presence of debt.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 presents the model solutions. Section 4 discusses the model's implications. Section 5 provides some suggestive empirical evidence consistent with the model predictions. Section 6 discusses further implications of our model and concludes.

#### 2. The model

We develop a continuous-time credit-risk model to illustrate the informational role of a firm's investment decision in a secondary asset market. All players in the model are risk-neutral and discount future consumptions at a constant rate of r.

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## 2.1 Firm assets and investment

The firm has assets that produce after-tax cash flows,  $x_t$ , over each time interval [t, t + dt). The corporate tax rate is  $\pi$ . The cash flow  $x_t$  is also interpreted as the firm's fundamental, asset quality, or type at time t. The cash flow evolves according to

$$\frac{dx_t}{x_t} = (\mu + \delta i_t)dt + \sigma dZ_t,\tag{1}$$

where  $\mu$  is the baseline growth rate,  $\sigma$  is the volatility, and  $Z_t$  is a standard Brownian motion representing random fluctuations in the firm's cash flows.  $\delta i_t$  is the incremental growth rate determined by the firm's investment decision  $i_t \in [0,1]$ . Specifically, at each time t, the growth rate of the cash flows increases by  $\delta i_t$  if the firm invests new capital of  $(gx_t + h)i_t$ , where g and h indicate the variable and fixed parts of the investment cost, respectively. Here, we consider linear investment technology for tractability. We assume  $\max\{g,h\} > 0$  to ignore a trivial case in which no costs are required for investment.

The investment decision is controlled by the firm's manager, who acts in the best interest of equityholders. Due to risk neutrality and linear technology, the manager invests either nothing or up to the maximum capacity. In equilibrium, the manager decides to invest when the firm's fundamental is larger than an endogenously determined investment threshold  $x_I$ . We verify the optimality of this threshold-type investment strategy later. To rule out another trivial case in which the manager never invests (that is,  $x_I = \infty$ ), we impose the following condition:

$$g < \frac{\delta(1-g)}{r-\mu-\delta}. (2)$$

That is, when g is not sufficiently large, the firm would invest when its fundamental is large enough. Under this condition, we first compute an optimal investment threshold of an unlevered firm, denoted by  $x_I^{FB}$ , in Appendix A.1. We call this threshold the first-best investment threshold.

The firm's fundamental,  $x_b$  is observed only by the firm manager. Meanwhile, the firm's investment decision,  $i_b$  is publicly observable. The investment decision can thus carry some information about the firm's fundamental because management uses a cut-off investment strategy. For clarification, we assume that the amount of actual investments,  $(gx_t + h)i_b$  is not publicly observable. Hence, outside investors cannot back out the firm's fundamental from the actual amount of investments. In practice, investments are mainly reported in financial statements as capital expenditures or R&D spending. These financial statements are, however, subject to accounting errors or manipulation. In this regard, we may alternatively assume that outside investors receive noisy or delayed reports on the amount of actual investment (or even on the cash flow level itself), as in Duffie and Lando (2001) and Benzoni et al. (2020). However, we do not pursue this task because (1) the learning effect is not a primary interest of this paper, and (2) the equilibrium outcome in the secondary asset market would not change as long as we focus only on separating equilibrium. Lastly, we further assume that the equity claims are not publicly traded in a secondary market, as do Duffie and Lando (2001) and Benzoni et al. (2020). Therefore, outsiders still cannot fully infer the firm's fundamental because the value of equity claims, potentially revealing the firm's fundamental, is not commonly observed.

# 2.2 Debt and default

The firm has one unit of outstanding perpetuity debt held by a representative debtholder. The debt claim promises to pay a coupon flow of c at each moment in time unless the firm defaults. The firm pays the coupons before making the investment decision. The coupon payment is tax deductible. When the firm fails to make the debt payment, the firm enters into bankruptcy. In bankruptcy, the debtholder acquires the firm's entire assets, and we assume the equityholders are wiped out.

The firm defaults for two reasons. The first type of default occurs due to a liquidity problem. Specifically, when the before-tax cash flows,  $\frac{x_t}{1-\pi}$ , are not enough to cover the coupon payment, c, the firm raises new funds from a pre-committed credit line, similarly as in He and Xiong (2012a) and Cheng and Milbradt (2012) [5]. Specifically, the firm can take out money from the preset credit line whenever needed. But the credit line is imperfect because the creditline provider, say, a bank, does not always have stand-by capital. We model this liquidity friction in a reduced form by assuming the firm fails to get the emergency funding with probability  $\phi dt$  in the Poisson manner. We let  $x_L = (1 - \pi)c$ , the threshold that triggers the liquidity shock. Once the firm has successfully paid the debt, using either the internal cash flows or the credit line, the manager makes the investment decision and then pays out all remaining cash flows as dividends. Here, without loss of generality, we assume the firm does not face liquidity problems when financing new investments [6].

The second type of default happens due to the limited liability of equityholders. That is, the firm optimally defaults when the equity value falls below zero. In our model, the equity value increases with the firm's fundamental, so the manager defaults when the cash flow level hits an endogenously determined default boundary  $x_D$ . We call this type of default event a fundamental-driven default. Here, note that in equilibrium,  $x_D$  must lie below  $x_L$  because (1) the before-investment net cash flow,  $x_t - (1 - \pi)c$ , is positive when the firm's fundamental is above  $x_L$  and (2) the option value of the investment is always positive.

For clarification, we assume the liquidity shock is publicly observable. So, outside investors can distinguish between liquidity-driven default and fundamental-driven default. Nevertheless, whether or not the liquidity shock is observable does not affect the equilibrium outcome. Because a firm at the fundamental-driven default boundary is the worst type among all default firms, the firm does not suffer from information asymmetry, no matter whether she can be identified from the reason of her default.

#### 2.3 Asset liquidation

We interpret bankruptcy generally to include both liquidation and reorganization. When default occurs, we assume that the investment opportunity completely expires, which reflects the bankruptcy costs to some extent, as in Diamond and He (2014) and Wong and Yu (2019). The intrinsic value of failed assets of type  $x \in [x_D, x_L)$  is thus simply given by  $\frac{x}{x-y}$ .

Further, we assume that the debtholder is not a good manager of a firm's assets. Due to this incompetency, the debtholder discounts the value of failed assets by a factor  $\alpha < 1$ . Meanwhile, potential asset buyers have relatively superior asset management skills because they discount the value of the same assets by a factor  $\beta$ , where  $\alpha < \beta \le 1$ . Potential buyers have deep pockets and behave competitively. Hence, the debtholder must prefer selling the entire assets to the potential buyers rather than keeping the assets in a frictionless economy.

These gains from trade are, however, hindered by information asymmetry in the secondary asset market. Specifically, when a firm defaults for the fundamental reason, the market can rationally infer that the firm's asset quality is  $x_D$ . So, in this case, information asymmetry is resolved. However, when a firm defaults due to a liquidity shock, its asset quality can take any value from the interval ( $x_D$ ,  $x_D$ ), observed only by the firm manager. More importantly, we assume that once the debtholder has taken control of the firm after default, the manager must

disclose the firm's inside information to the debtholder. We believe this assumption is reasonable because, for instance, in the United States, creditors and bankruptcy trustees can ask corporate management or debtors questions under oath regarding matters related to the bankruptcy case through the 341 hearing and the Bankruptcy Rule 2004 Examination. Alternatively, we may assume that the current management handles the bankruptcy process on behalf of the debtholder. This assumption also makes sense because a sizable number of existing CEOs stay with their bankrupt firms for a certain period of time, as mentioned in the introduction. Under either of the assumptions, the debtholder's enhanced accessibility to the firm's information leads to information asymmetry in the secondary asset market.

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In the presence of asymmetric information, the debtholder chooses to retain a partial fraction of the assets to signal their true quality, similarly as in Leland and Pyle (1977) [7]. Let  $f(x) \in [0, 1]$  denote the fraction of the assets retained by the debtholder of a firm of type  $x \in [x_D, x_L)$ . A key feature of our model is that the market forms the beliefs on the asset quality not only from the debtholder's asset retention decision but also from the firm's investment decision before default. An equilibrium price of the assets, p(x), is then set by the market's beliefs in the break-even manner. Accordingly, for any pair of f(x) and p(x), the recovery value of a firm of type x is equal to

$$R(x) = \underbrace{f(x) \frac{\alpha x}{r - \mu}}_{\text{from retention}} + \underbrace{(1 - f(x))p(x)}_{\text{from liquidation}}.$$
(3)

The first term is the debtholder's own valuation of the retained portion of the assets, and the second term is the proceeds received from the remaining portion of the assets liquidated. Throughout the paper, we focus on a separating equilibrium in which the asset quality is truthfully revealed in the secondary asset market. Extending our analysis to allow for pooling equilibrium complicates the model since we then need to introduce a stationary distribution of firm types, but this does not change the qualitative implications of our model.

#### 3. Model solutions

# 3.1 Liquidation value with signaling

We first analyze the signaling game in the secondary asset market, taking any default and investment thresholds,  $x_D$  and  $x_I$ , such as  $x_D < x_I$ , as given. We can ignore a special case of  $x_D = x_I$  because that case does not occur in equilibrium. As mentioned above, we focus on a separating equilibrium in which the asset types are fully revealed. In this equilibrium, the liquidated portion of assets is fairly priced according to the break-even condition for potential buyers. That is, for any type x, we have  $p(x) = \frac{\beta x}{r - \mu}$ . Formula (3) then implies that the debtholder of a type-x firm recovers

debtholder of a type-
$$x$$
 firm recovers
$$R(x) = f(x) \frac{\alpha x}{r - \mu} + (1 - f(x)) \frac{\beta x}{r - \mu} \tag{4}$$

in total in bankruptcy. We hereafter refer to the debtholder of a type-x firm as the debtholder of type x.

To solve for the equilibrium asset retention ratio f(x), consider two cases separately: (1)  $x_I < x_L$  and (2)  $x_L \le x_I$ . In what follows, we show that

$$f(x) = \begin{cases} 1 - \left(\frac{x_D}{x}\right)^{\frac{\beta}{\beta - \alpha}}, & \forall x \in [x_D, x_I) \\ 1 - \left(\frac{x_I}{x}\right)^{\frac{\beta}{\beta - \alpha}}, & \forall x \in [x_I, x_L) \end{cases}$$
(5)

and

$$f(x) = 1 - \left(\frac{x_D}{x}\right)^{\frac{\beta}{\beta - \alpha}}, \quad \forall x \in [x_D, x_L)$$
 for cases (1) and (2), respectively. (6)

Case (1).  $x_I < x_I$ . In this case, potential buyers can infer whether a firm's type belongs to  $[x_D, x_I]$  or  $[x_I, x_L]$  based on the firm's past investment decision. Hence, the debtholders of types  $x \in [x_D, x_I)$  do not need to differentiate themselves from the debtholders of types  $y \in [x_I, x_I)$ , and vice versa. We first consider the low-quality group, the group of debtholders whose types belong to  $[x_D, x_I]$ . Suppose the debtholder of type  $x \in [x_D, x_I]$  mimics another debtholder of type  $y \in [x_D, x_I]$ . Then the recovery value to the former debtholder will be

$$f(y)\frac{\alpha x}{r-\mu} + (1-f(y))\frac{\beta y}{r-\mu}.$$

Hence, we can write the maximization problem of that debtholder as

$$\max_{y \in [x_D, x_I)} f(y)\alpha x + (1 - f(y))\beta y, \tag{7}$$

ignoring the common term  $1/(r-\mu)$ . In a separating equilibrium, every debtholder truthfully reveals her type. So, for that equilibrium to exist, the following first-order condition, evaluated at x itself, must hold:

$$\underbrace{(\beta - \alpha)xf'(x)}_{\text{marginal cost}} = \underbrace{\beta(1 - f(x))}_{\text{marginal benefit}} \forall x \in [x_D, x_I). \tag{8}$$

The left-hand side indicates the marginal cost of deviation in the sense that when the debtholder of type x mimics the type  $x + \Delta$ , the deviating debtholder has to retain an additional fraction  $f(x)\Delta$  of the assets instead of selling that part of the assets. The right-hand side denotes the marginal benefit of deviation in that the selling price of assets increases by  $\beta\Delta$  due to mimicking. The boundary condition for equation (8) is  $f(x_D) = 0$  because the debtholder of the worst type chooses to sell the entire fraction of assets. The unique solution to the equation is then given by the first line in (5). Here, note that the first-order condition (8) is a necessary condition for optimality. In Appendix A.2, we show that f(x) derived above is globally optimal.

Applying the same argument, we can compute the asset retention ratios for the debtholders whose types belong to  $[x_l, x_l]$ . We call this group of debtholders the high-quality group. Notably, because  $x_I$  is the worst type in this group, the debtholder of type  $x_I$  can sell the entire fraction of assets. So, to characterize an equilibrium for this group, we need to solve equation (8) over the interval  $[x_i, x_l]$  subject to  $f(x_l) = 0$ . The solution is given by the formula in the second line in (5).

The left panel in Figure 1 plots the retention ratio f(x) for every  $x \in [x_D, x_L)$ . Within each group, the retention ratio f(x) increases in x, meaning that a debtholder of a higher type retains more of her assets for signaling purposes. But f(x) jumps down at  $x_I$  because the debtholder of type  $x_I$  no longer needs to differentiate herself against the debtholders in the low-quality group.

Case (2).  $x_I \ge x_L$ . In this case, potential buyers cannot separate defaulted firms into two subgroups based on their past investment decisions. Hence, all debtholders, whose types range over  $[x_D, x_L)$ , need to differentiate themselves from each other. So, in this case, we need to solve equation (8) over the entire interval  $[x_D, x_L)$  subject to  $f(x_D) = 0$ . The solution is given by the formula in (6), which merely increases in x over the entire interval as shown in the right panel in Figure 1.

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The main insight behind the above results is that in case (1), firms' investment decisions carry information to the secondary asset market. The fact that the investment threshold lies inside the liquidity-driven default region means that illiquid firms make differentiated investment decisions before default. As a result, potential buyers can learn partial information about the asset quality of those firms from their past investment decisions. This piece of information is valuable because the degree of inefficient asset retention is reduced due to that information. Figure 1 clearly illustrates this effect: the asset retention ratio in case (1) is always lower than that in case (2) in which the investment decision carries no informational value as illiquid firms make the same investment decision.

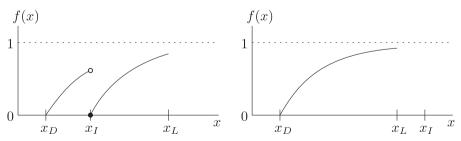
#### 3.2 Debt valuation

This section evaluates the present value of debt, D(x). For any default and investment thresholds, D(x) satisfies the following ordinary differential equation:

$$rD(x) = c + \phi 1_{x < x_L} (R(x) - D(x)) + (\mu + \delta 1_{x \ge x_I}) x D_x(x) + \frac{\sigma^2}{2} x^2 D_{xx}(x)$$

subject to  $D(x_D) = \frac{\beta x_D}{r_- \mu}$ . The term on the left-hand side is the required return on debt. The first term on the right-hand side is the coupon payment. The second term indicates the liquidity-driven default event, where R(x) is the recovery value of the firm. The remaining terms explain the expected change in the debt value due to the time-varying cash flows. The boundary condition denotes the recovery value of a firm when a fundamental-driven default occurs. This recovery value is fairly priced because of rational expectation. The above equation says that the firm's investment decision enters into debt valuation through two channels. First, the investment decision affects the probability of default by determining the growth rate of the firm's cash flows. Second, the investment decision alters the recovery value of debt, R(x), by changing the information structure in the secondary asset market.

We provide the solution for D(x) in closed form here. Let  $\xi = \frac{\beta}{\beta - \alpha}$ . Then, in the case of  $x_I < x_L$ , we have



**Note(s):** The left panel corresponds to the case where  $x_I < x_L$ . The right panel corresponds to the case where  $x_L \le x_I$ 

**Source(s):** Figure by authors

Figure 1. The asset retention ratio f(x)

$$D(x) = \begin{cases} \frac{c}{r+\phi} + \frac{\phi \alpha x}{(r+\phi-\mu)(r-\mu)} + \frac{\phi(\beta-\alpha)x_D^{\varepsilon}x^{1-\varepsilon}/(r-\mu)}{r+\phi-\mu(1-\xi) + \frac{\sigma^2}{2}(1-\xi)\xi} + A_1x^{\zeta_1} + A_2x^{\zeta_2}, & \forall x \in [x_D,x_I] \\ \frac{c}{r+\phi} + \frac{\phi \alpha x}{(r+\phi-\mu-\delta)(r-\mu)} + \frac{\phi\theta(\beta-\alpha)x_I^{\varepsilon}x^{1-\varepsilon}/(r-\mu)}{r+\phi-(\mu+\delta)(1-\xi) + \frac{\sigma^2}{2}(1-\xi)\xi} + A_3x^{\zeta_3} + A_4x^{\zeta_4}, & \forall x \in [x_I,x_L] \\ \frac{c}{r} + A_5x^{\zeta_5}, & \forall x \in [x_L,\infty) \end{cases}$$

where the constants,  $\{A_1, \ldots, A_5, \zeta_1, \ldots, \zeta_5\}$ , are given in Appendix A.3. In the case of  $x_1 \ge x_L$ , we have

$$D(x) = \begin{cases} \frac{c}{r+\phi} + \frac{\phi\alpha x}{(r+\phi-\mu)(r-\mu)} + \frac{\phi(\beta-\alpha)x_D^{\epsilon}x^{1-\epsilon}/(r-\mu)}{r+\phi-\mu(1-\xi) + \frac{\sigma^2}{2}(1-\xi)\xi} + A_1x^{\epsilon_1} + A_2x^{\epsilon_2}, & \forall x \in [x_D,x_L] \\ \frac{c}{r} + A_3x^{\epsilon_3} + A_4x^{\epsilon_4}, & \forall x \in [x_L,x_I] \\ \frac{c}{r} + A_5x^{\epsilon_5}, & \forall x \in [x_L,\infty), \end{cases}$$

where the constants,  $\{A_1, \ldots, A_5, \zeta_1, \ldots, \zeta_5\}$ , are provided in Appendix A.3.

# 3.3 Equity valuation

The equity value, E(x), satisfies the following Hamilton-Jacobi-Bellman equation:

$$rE(x) = \max_{i \in [0,1]} x - (1-\pi)c - \phi 1_{x < x_L} E(x) - i(gx+h) + (\mu + \delta i)x E_x(x) + \frac{\sigma^2}{2} x^2 E_{xx}(x)$$
 (9)

subject to  $E(x_D) = E_x(x_D) = 0$ . The left-hand side is the required return on equity. The first two terms on the right-hand side are the net cash flow before investment. The third term indicates the liquidity-driven default at which equity is wiped out. Regarding the investment decision, the equityholders invest up to the maximum if  $\delta x E_x(x) > gx + h$  and do not make any investment if  $\delta x E_x(x) < gx + h$ . When  $\delta x E_x(x) = gx + h$ , the equityholders are indifferent between investing and not investing. In Appendix A.4, we prove the global optimality of the threshold-type investment strategy. The remaining terms in (11) explain the expected change in the equity value due to the time-varying cash flows. The boundary condition  $E(x_D) = 0$  is the value-matching condition that sets the equity value to zero in default. The other condition,  $E_x(x_D) = 0$ , the smooth-pasting condition, is the optimality condition for default.

For any given thresholds  $x_D$  and  $x_D$  a closed-form solution for the equity value is given below. In the case of  $x_I < x_I$ , we have

$$E(x) = \begin{cases} \frac{-(1-\pi)c}{r+\phi} + \frac{x}{r+\phi-\mu} + B_1 x^{\zeta_1} + B_2 x^{\zeta_2}, & \forall x \in [x_D, x_I] \\ \frac{-(1-\pi)c - h}{r+\phi} + \frac{(1-g)x}{r+\phi-\mu-\delta} + B_3 x^{\zeta_3} + B_4 x^{\zeta_4}, & \forall x \in [x_I, x_L] \\ \frac{-(1-\pi)c - h}{r} + \frac{(1-g)x}{r-\mu-\delta} + B_5 x^{\zeta_5}, & \forall x \in [x_L, \infty), \end{cases}$$

where the constants,  $\{B_1, \ldots, B_5, \zeta_1, \ldots, \zeta_5\}$ , are given in Appendix A.4. In the case of  $x_I \ge x_L$ , we have

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$$E(x) = \begin{cases} \frac{-(1-\pi)c}{r+\phi} + \frac{x}{r+\phi-\mu} + B_1 x^{\xi_1} + B_2 x^{\xi_2}, & \forall x \in [x_D, x_L] \\ \frac{-(1-\pi)c}{r} + \frac{x}{r-\mu} + B_3 x^{\xi_3} + B_4 x^{\xi_4}, & \forall x \in [x_L, x_I] \\ \frac{-(1-\pi)c - h}{r} + \frac{(1-g)x}{r-\mu-\delta} + B_5 x^{\xi_5}, & \forall x \in [x_I, \infty), \end{cases}$$

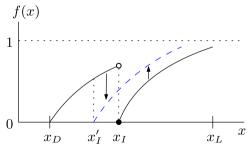
where the constants,  $\{B_1, \ldots, B_5, \zeta_1, \ldots, \zeta_5\}$ , are provided in Appendix A.4.

Given the above closed-form solution, we solve for the optimal thresholds  $x_D$  and  $x_I$  numerically, using the optimality conditions,  $E_x(x_D) = 0$  and  $\delta x_I E_x(x_I) = g x_I + h$ . Note that a trivial equilibrium with  $x_I = x_D$  does not exist as long as gh > 0.

# 3.4 Informational role of investment

To emphasize the intuition for the informational role of investment, we analyze the effect of a change in the investment threshold  $x_I$  on the secondary asset market, keeping the endogenous default boundary  $x_D$  fixed [8]. We focus on the case of  $x_I < x_L$  because firms' investment decisions carry an informational value only when firms, whose fundamentals lie over the default region  $[x_D, x_L)$ , make differentiated investment decisions before default. Figure 2 depicts the effect of a decrease in the investment threshold on the asset retention ratio, where  $x_I$  and  $x_I'$  denote the old and new investment thresholds, respectively. We investigate this effect by dividing the firms over the default region into three subgroups.

First, the asset retention ratios of firms whose types belong to  $[x_D, x_I')$  remain the same. This result was obtained because (1) these firms do not invest even after the investment threshold is decreased from  $x_I$  to  $x_I'$  and (2) the worst asset quality among the group of noninvesting firms stays fixed at  $x_D$ . A small decrease in the investment threshold does not have any informational effect on severely bad firms.



**Note(s):** The solid curve plots the original asset retention ratio. The dashed curve plots the new asset retention ratio. Here, the dashed curve over the interval  $x \in [x_D, x_J']$  is omitted because the asset retention ratio over that interval remains unchanged

**Source(s):** Figure by authors

Figure 2. This figure depicts how the asset retention ratio changes when the investment threshold decreases from  $x_t$  to  $x'_t$ 

Second, the asset retention ratios of firms whose types belong to  $[x'_I, x_I)$ , decrease. Specifically, when the investment threshold is  $x_I$ , these firms do not invest and thus, they are pooled together with other non-investing firms, whose asset quality is even lower than  $x'_I$ . So, the debtholders whose types are between  $x'_I$  and  $x_I$  retain a significant portion of their assets in default to signal the asset quality. However, when the investment threshold decreases to  $x'_I$ , those firms whose types lie in  $[x'_I, x_I)$  now choose to invest, and so they are grouped together with other investing firms with asset quality higher than  $x_I$ . As a result, the debtholders of those newly investing firms can sell a larger portion of their assets to potential buyers, which is a more efficient outcome compared to the case where the investment threshold is  $x_I$ .

Third, the asset retention ratios of firms whose types belong to  $[x_I, x_L)$  increase. As mentioned above, when the investment threshold goes down from  $x_I$  to  $x_I'$ , firms whose types lie in  $[x_I', x_I)$  now decide to invest. Accordingly, these relatively low-quality firms are added to the group of investing firms. So, the debtholders of type  $x \in [x_I, x_I)$ , whose firms decided to invest even before the investment threshold is reduced, must retain a larger portion of their assets to differentiate themselves against the newly added low-quality firms. The main interest of our paper lies in this spillover effect. That is, when more relatively low-quality firms start to invest, the recovery values of relatively high-quality firms will be lowered. We provide empirical support for this result in Section 5.

The three results above say that the overall effect of a change in the investment threshold on liquidity in the secondary asset market would be ambiguous. Nonetheless, we can show that the overall effect must be non-monotone in the investment threshold. When  $x_I$  is sufficiently close to  $x_D$ , almost all firms choose to invest, so firms' investment decisions barely mitigate the information friction in the secondary asset market. When  $x_I$  is sufficiently close to or even higher than  $x_L$ , almost all firms choose not to invest. Thus, the secondary asset market again enjoys few benefits from the informational value of investment. Therefore, the overall informational effect must be maximized in the interior case in which the investment threshold lies at some point between  $x_D$  and  $x_L$ .

#### 4. Model implications

In this section, we explore the quantitative implications of our model. We first estimate the model parameter values. We then analyze the effects of investment cost and debt overhang on liquidity in a secondary asset market and debt and firm values.

# 4.1 Baseline parameter values

Table 1 summarizes the baseline parameter values. We set the risk-free rate r to 5%, the tax rate  $\pi$  to 25%, and the asset volatility  $\sigma$  to 15% as standard in the literature (e.g. Miao (2005), Hackbarth *et al.* (2006), Chen *et al.* (2018), and Wong and Yu (2019)). We set the baseline

Definition	Value	Definition	Value
Risk-free rate Asset volatility Incremental productivity Fixed the investment cost Discount factor of asset buyers Coupon payment Source(s): Table by authors	r = 5% $\sigma = 15\%$ $\delta = 2.5\%$ h = 0.2 $\beta = 80\%$ c = 1.76	Corporate tax rate Baseline growth rate Proportional investment cost Discount factor of debtholders Intensity of liquidity shock	$\pi = 25\%$ $\mu = 1\%$ $g = 0.1$ $\alpha = 40\%$ $\phi = 1$

**Table 1.**Baseline Parameter Values

growth rate  $\mu$  as 1%, and the incremental growth rate  $\delta$  as 2.5%, similarly as in Diamond and He (2014) and Wong and Yu (2019). This choice roughly reflects the fact that the growth rate of aggregate corporate profits lies between 1 and 4.5% (see Chen *et al.* (2018)).

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Regarding the investment costs, we set g and h as 0.1 and 0.2, respectively, consistent with Studies: 선물연구 the values in Diamond and He (2014) and Wong and Yu (2019). Under this parameter choice, the fraction of earnings reinvested by a firm ranges between 10 and 30.7%. We set the discount factor of debtholders,  $\alpha$ , as 40% and that of asset buyers,  $\beta$ , as 80%. These numbers are along with the fact that the recovery rate of corporate bonds ranges from 40% to 80% (see Hackbarth et al. (2006), Chen (2010), Chen et al. (2018)). We set the intensity of the liquidity shock,  $\phi$ , as 1, which means that the average survival time of a firm exposed to a liquidity shock is 1 year if the firm's fundamental were to be fixed over time. This choice is consistent with the structural parameter estimates by Schroth et al. (2014). We choose the initial cash flow level  $x_0$  as 1. In our model, the scale-invariance property does not hold because of the fixed component of the investment cost. Nonetheless, our results do not heavily rely on the choice of  $x_0$  unless chosen to be far away from 1. Lastly, we use the credit rating symbols defined by Moody's and choose the coupon size c to target the average credit spread of speculative Ba-rated bonds, following the approach by He and Xiong (2012b) and He and Milbradt (2014). Specifically, according to Rossi (2014), the credit spread of Ba-rated bonds with maturity above 10 years is 403 basis points.

Our model generates this target number under the choice of c = 1.76 [9]. When considering the firms with other credit ratings below, we choose different  $x_0$  appropriately, fixing the coupon size to 1.76. Under this set of parameters, we have  $x_D = 0.805$ ,  $x_I = 0.966$ , and  $x_L = 1.317$  in equilibrium.

# 4.2 The effects of investment cost

In this section, we study the comparative-static result with respect to the investment cost. Note that decreasing the investment cost can be interpreted as a fiscal policy measure that provides tax-based incentives for new investments. One example of that policy is the provision of bonus depreciation that the US government offered during the 2001 and 2008 recessions. Our model introduces a new information-based mechanism through which such a stimulus program can adversely affect firm and debt value.

In our model, the investment cost affects debt value through two channels. On the one hand, a decrease in investment cost reduces a firm's default risk, which we call the default-risk channel. When the investment cost is lowered, a firm is more likely to invest. The dashed curves in Panels A and B of Figure 3 show this positive relation between both variable and fixed costs of investment, that is, g and h, and the investment threshold  $x_I$ . As a result, the firm's cash flow grows at a higher rate, resulting in a lower probability of default. Moreover, the firm delays the timing of default because a decrease in investment cost makes it more valuable. This effect reduces the default probability further. The solid curves in the two panels demonstrate the positive relation between the investment cost and the default threshold  $x_D$ .

On the other hand, investment cost affects the recovery value of debt in default through the informational channel. As mentioned above, when investment cost is reduced, a firm's incentive to invest increases, which means the investment threshold decreases. The decrease in the investment threshold can then change the recovery value of debt in any direction by affecting the beliefs of potential buyers in a secondary asset market, as discussed in Section 3.4.

As a reduction in investment cost can change debt value in a non-monotone way, especially due to the informational channel, we now examine the effects of investment cost more thoroughly, considering firms with different credit ratings. This analysis will show that



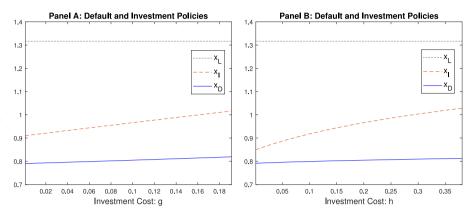


Figure 3.
The effects of investment cost on the default and investment thresholds

**Note(s):** Panel A plots the effects of the variable cost of investment. Panel B plots the effects of the fixed cost of investment

Source(s): Figure by authors

the adverse effects of the above-mentioned fiscal policy are more likely to occur to speculative Ba-rated firms rather than to investment-grade or even more speculative firms.

4.2.1 Case for the Ba-rated firm. Figure 4 depicts the effects of investment cost on the Ba-rated firm whose asset quality has been set to  $x_{Ba} := 1$  in the above calibration. According to Panel A of Figure 3, the investment threshold  $x_I$  exceeds  $x_{Ba}$  when the variable cost of investment, g, is higher than 0.161. This result means that if g is lower than 0.161, the Ba-rated firm belongs to the high-quality group of firms that make investments and face low default risk; otherwise, the Ba-rated firm belongs to the low-quality group of firms that do not make investments and face high default risk. So, when g is lower than 0.161, a decrease in the investment cost must reduce the recovery value of debt in default through the informational channel, as discussed in Section 3.4. In addition, since the default risk is low in this case, the

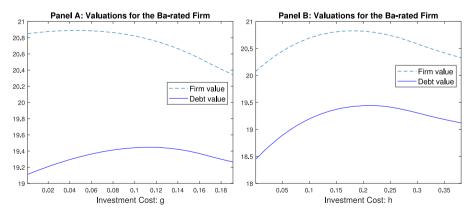


Figure 4.
The effects of investment cost on the debt and firm values for the Ba-rated firm

**Note(s):** Panel A plots the effects of the variable cost of investment. Panel B plots the effects of the fixed cost of investment

Source(s): Figure by authors

positive effect of reducing the investment cost through the default-risk channel is small. Hence, when *g* is low, a decrease in the investment cost is more likely to hurt the debt value. Panel A of Figure 4 shows that the debt value decreases in the investment cost *g* when that cost is below 0.115. Specifically, when *g* decreases from 0.1 to 0, debt value decreases by 1.8%.

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However, we show that when g is high, debt value no longer increases in the investment cost. Specifically, when the investment cost is so high that a firm belongs to the low-quality group, a marginal decrease in the investment threshold does not affect the recovery value of debt through the informational channel, as discussed in Section 3.4. However, the firms in the low-quality group face higher default risk. So, when g is high, debt value is more likely to decrease in the investment cost because the default-risk channel tends to dominate the informational channel in this case. Panel A of Figure 4 demonstrates this negative relation between investment cost and debt value for the case where g is higher than 0.115.

According to the same panel, firm value increases in the investment cost g over the interval [0, 0.048], smaller than [0,0.115]. This result says that the effect of reducing the investment cost on firm value would more likely be positive compared to the effect on debt value. The underlying reason is simple: a decrease in investment cost always increases equity value through both the NPV channel and the default-risk channel. Here, the NPV channel means that a reduction in investment cost directly saves the costs that equityholders need to bear when making new investments. Nonetheless, as g decreases, the benefit of a reduction in investment cost on firm value is lessened. Eventually, it turns negative because of the adverse informational effect on debt value. Specifically, when g decreases from 0.15 to 0.1, firm value increases by 1.1%; when g decreases from 0.1 to 0.05, firm value increases by 0.3%; and when g decreases from 0.05 to 0, firm value decreases by 0.2%. Put simply, firm value exhibits a g-shaped pattern.

Panel B of Figure 4 plots the effect of the fixed cost of investment, h, on the debt and firm values. The graph shows that debt value increases in h over the interval [0, 0.201], while firm value increases in h over the interval [0,0.179]. Moreover, when h decreases from 0.2 to 0.1, debt value decreases by 1.3%, and firm value decreases by 0.6%. These results show that debt and firm values can decrease as investment cost is reduced, especially when that cost is low, regardless of whether we change the variable or fixed investment cost.

One interesting observation is that the negative effect on debt or firm value would be more significant when we reduce the fixed cost rather than the variable cost. Intuitively, the investment decision of firms with poor fundamentals is more sensitive to changes in the fixed cost than to changes in the variable cost. Also, under the baseline parameter values, the fundamental of the marginal firm, which is indifferent between investing and not investing, is low enough; that is,  $x_I$  is lower than the fundamental of the speculative Ba-rated firm. Hence, under the baseline parameter values, the investment threshold must respond more sensitively to changes in the fixed cost. As a result, reducing the fixed cost is more likely to cause sizable negative effects on liquidity in the secondary asset market than the variable cost does, driving the above result.

4.2.2 Case for the firms with other ratings. Panels A and B of Figure 5 plot the comparative statics results for the A-rated firm. We omit to consider other investment-grade firms because the results for those firms are similar. For the A-rated firm, we set the asset fundamental to  $x_A := 1.40$ . Specifically, according to Rossi (2014), the average credit spread of the A-rated bonds with maturity above 10 years is 164 bps. In our model, a firm with a cash flow level of 1.40 generates this target spread under the baseline parameter values.

The dashed curves in the panels show that firm value merely decreases with both variable and fixed costs of investment. Note that  $x_A = 1.40$  lies away from the default region  $[x_D, x_L] = [0.80, 1.32]$ . So, the informational effect of a change in the investment cost must be small compared to the direct NPV effect of that change. As a result, the total value of the A-rated firm generally decreases in the investment cost.



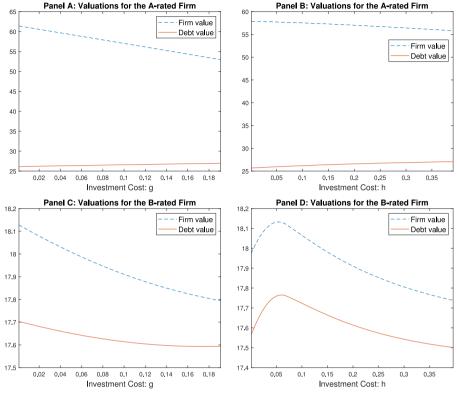


Figure 5.
The effects of investment cost on the debt and firm values for the A-rated and B-rated firms

**Note(s):** Panels A and B plot the effects of g and h for the A-rated firm, respectively.

Panels C and D plot the effects of g and h for the B-rated firm, respectively

Source(s): Figure by authors

Meanwhile, the solid curves show that debt value decreases when investment cost is reduced, as for the Ba-rated firm. This result makes sense because a change in investment cost does not cause the direct NPV effect on debt value but causes an informational effect for the A-rated firm as the firm's fundamental fluctuates. Hence, the debt value of the A-rated firm can also decrease when the investment cost decreases. Here, the debt value increases in h over a wider interval, compared to the case of the Ba-rated firm, simply because  $x_A > x_{Ba}$ . Although omitted in the graph, the debt value will decrease if we extend the x-axis further so that  $x_I$  exceeds  $x_L$ .

Panels C and D of Figure 5 depict the effects of the investment cost for the B-rated firm. The results for other extremely speculative firms are similar. According to Rossi (2014) again, the average credit spread of the B-rated bonds with maturity above 10 years is 510 bps. To match this target spread, we set the asset fundamental of the B-rated firm as  $x_B = 0.91$ . In Section 3.4, we have shown that a marginal decrease in the investment threshold causes either zero or positive effects for the firms in the low-quality group. Hence, combined with the positive effects through the NPV and default-risk channels, a reduction in investment cost generally leads to an increase in both debt and firm values for the B-rated firm, as seen in the two panels. As an exception, when h is sufficiently small, both debt and firm values increase in h because the B-rated firm belongs to the high-quality group.

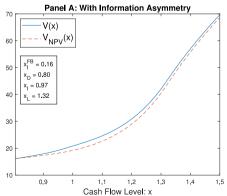
# 4.3 The effects of debt overhang

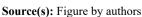
In our model, management makes the investment decision to maximize equity value. Therefore the classic debt overhang problem arises: equityholders may forgo a positive-NPV project because most of the investment benefits accrue to debtholders. According to the literature on debt overhang, following Myers (1977), the presence of debt overhang has a negative impact on investment activity and firm value. So, common wisdom suggests that to maximize firm value, a firm should follow the simple NPV rule when making investment decisions.

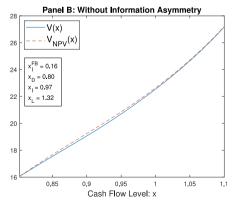
We challenge this conventional view by adding the informational role of investment into the analysis. Because of the publicly observable nature of investment, firms' investment decisions affect firm value not only through the NPV channel but also through the informational channel. In our model, the NPV rule corresponds to the strategy the manager invests if and only if  $x_t \ge x_I^{FB}$ , where  $x_I^{FB}$  is the first-best investment threshold mentioned above. Here, we assume that when following the NPV rule, the manager uses the same default threshold  $x_D$  obtained in the original model. When  $x_I^{FB}$  is placed below  $x_D$ , the manager always invests. This approach, commonly used in the literature, allows us to extract only the pure effect of debt overhang (Hackbarth and Mauer, 2012; Chen and Manso, 2017). We denote the firm value computed under the NPV rule by  $V_{NPV}(x)$ .

The solid curve in Panel A of Figure 6 denotes the original firm value V(x), while the dashed curve indicates the firm value obtained under the NPV rule,  $V_{NPV}(x)$ . The graphs show that, under the baseline parameters, the value of a firm is larger when management maximizes equity value rather than follows the NPV rule, not only for the firm with fundamental  $x_0 = 1$  but also for all other firms. Intuitively, changing the investment strategy from the one maximizing equity value to the NPV rule eliminates the under-investment problem brought by debt overhang. This effect can be seen from  $x_1^{FB} < x_I$ .

However, such a decrease in the investment threshold can cause two opposite effects on firm value. On the one hand, firm value can increase as more positive-NPV projects are undertaken. On the other hand, a decrease in the investment threshold can reduce the informational value of investment, thereby hurting the firm's recovery value in default. The above numerical result says that the latter effect dominates the former effect under reasonable parameter values. Hence, our model predicts that firm value would be lower if management adopts the NPV rule, especially when the secondary asset market suffers from severe information asymmetry. In this regard, our model sheds new light on the positive role







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Figure 6. The effects of debt overhang on firm value. Panel A considers the economy with information asymmetry. Panel B considers the economy without information asymmetry. In both panels, the solid curve plots the value of a firm whose manager maximizes equity value, whereas the dashed curve plots the value of a firm whose manager follows the NPV rule

of debt overhang friction. In sum, the agency friction between equity and debt can mitigate information friction in the secondary asset market.

The positive role of debt overhang exists because there is information friction in the secondary asset market. In an economy with full information, the firm value is always higher if the manager follows the NPV rule, as shown in Panel B of Figure 6. In this case, because potential buyers are fully informed of a firm's asset quality, the investment decision does not create any informational value. Therefore, the presence of debt overhang only negatively affects the firm value by destroying positive-NPV projects.

# 5. Suggestive empirical evidence

This section presents suggestive empirical evidence that accords with the model's predictions. Ideally, to test our model predictions, we would like to examine the relation between firm value and investment cost. However, this empirical exercise would be challenging because investment cost is difficult to measure, and firm value can increase with leverage due to other channels that are hard to disentangle empirically. Therefore, we instead look at the model's prediction about firms' investment rates and recovery values.

In Section 3.1, we find that a decrease in investment threshold leads to more investment by firms with low asset quality and, in the meantime, lower recovery value for firms with high asset quality. Therefore, the model generates an informational spillover between firms with different types and predicts a negative correlation between low-type firms' investment rate and high-type firms' recovery value. Although our model predictions apply to any imperfect predictor of firm quality, in the empirical exercise, we estimate the correlation between the investment rate of firms with low credit ratings and the recovery rate of firms with high credit ratings. We interpret firms with low private types in the model as firms with lower credit ratings. This interpretation will not affect our qualitative results as credit ratings are imperfect signals of firms' private types. For example, such an interpretation may more readily apply to firms in China, where despite rising default rates in 2019, as of the end of May 2019, over 50% of outstanding corporate bonds had an AAA rating, compared to about 6% in the United States. More specifically, if we assume knowing the rating of a firm does not change the market's belief about the support of the asset quality distribution, then the introduction of rating information does not change our analysis of the model. When the support of the asset quality distribution is fixed, firms' asset retention behaviors fully reveal firm types in a separating equilibrium, rendering the rating information irrelevant.

To empirically study the informational spillover of different firms on recovery rates, we use data on the investment rates, recovery rates, and credit ratings from Compustat and Moody's Default and Recovery Database from 1985 to 2015. More details on the sample and variable construction can be found in Appendix A.5. In particular, we test whether the correlation between the investment by low-rated firms and the recovery value of high-rated firms is negative. We focus on non-investment grade firms with Ba and B credit ratings for which we expect the magnitude of information asymmetry to be higher [10].

Using observations at the credit rating-year level, we estimate the following empirical specification to test the informational spillovers of differently ex-ante-rated firms:

Recovery rate<sub>c,t</sub> = 
$$\alpha + \beta_1$$
Investment rate<sub>c,t-2</sub> +  $\beta_2$ Investment rate<sub>c-1,t-2</sub> +  $\varepsilon_{c,t}$ , (10)

where t indexes a year and c indexes a credit rating ranked descending from high to low, so c-1 is the credit rating immediately below c. The explained variable is the average recovery rate of senior unsecured bonds with a credit rating of c at year t. Two explanatory variables are used in the regression. The first is the average net tangible capital investment rate at year t-2 across the firms with a crediting rating c. The second explanatory variable is the

average net investment rate at year t-2 across the firms with a credit rating c-1. That is, if cis a Ba rating, then c-1 is a B rating. The investment rates are defined as the change in net plants, property, and equipment divided by the previous fiscal year's total asset value. We focus on tangible investments rather than intangible investments such as R&D because Studies: 선물연구 investments in tangible capital are more observable by outsiders. We use the investment rates at year t-2 to account for the fact that investment in tangible capital usually takes more than one year to complete. The two explanatory variables are to horse race whether investment rates and recovery rates are related and whether this relation is driven by lowerrated firms or equally-rated firms. To account for overlapping observations and autocorrelations in the aggregate variables, we use the Newey-West correction with 3-year lags to estimate the standard errors for the regression coefficients.

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In column (1) of Table 2, we find that even after controlling for the investment rate of Barated firms, a higher investment rate of B-rated firms predicts a lower recovery rate of relatively high-rated firms. In this regression, we focus on the firms rated Baa or below because firms rated A and above rarely default and may have less information asymmetry, making them less irrelevant to our model predictions,[11] Fixing the investment rate of Ba firms, a 1% higher investment rate of B firms is related to a -0.77% lower recovery rate of Ba firms that default two years later. The economic impact of B-rated firm investment affects the recovery rates of Ba firms more than their own investment rates. In column (2), unconditionally, a 1% higher average of B firm net investment rate is related to a 0.87% decrease on average. Figure 7 visualizes this negative relationship, which does not appear to be driven by any particular year. Meanwhile, column (3) shows the own-rating relationship between investment rates and recovery rates for B-rated firms is negative but not statistically significant. This finding is consistent with our model's prediction that more investment in low-rated firms can reduce the informational value contained in the investment decision, leading to a decrease in the recovery rate of high-rated firms.

While the negative relation between investment rate and recovery rate may also appear consistent with alternative models, we argue that alternative models would not produce the cross-correlations observed. For example, over-investment in negative-NPV projects can lead to a negative correlation between investment and recovery rates. In reality, firms may invest in negative-NPV projects for several reasons. First, the well-known risk-shifting problem can cause inefficient investments in inefficient projects (Jensen and Meckling, 1976; Eisdorfer, 2008: Jiang and Kim, 2021). Second, overconfident managers can also invest in negative-NPV

Explained variable =	Ba recovery rate (1)	(2)	B recovery rate (3)
Ba Investment Rate	-0.288***		
B Investment Rate	(0.109) -0.771***	-0.877***	-0.457
Constant	(0.164) 60.915***	(0.157) 57.073***	(0.318) 56.786***
Observations	(4.111)	(4.703)	(3.693) 27
Observations $R^2$	22 0.316	22 0.297	0.095

Note(s): The data on the investment rates, credit ratings and recovery rates come from Compustat and Moody's Default and Recovery Database. Details about the explained and explanatory variables are described in Appendix A.5. We use Newey-West standard errors permitting up to 3 lags of autocorrelation. \* signifies p < 0.1, \*\* signifies p < 0.05, and \*\*\* signifies p < 0.01

Source(s): Table by authors

Table 2. This table presents the coefficients and standard errors of the regression described in (10) over the period from 1984 to 2015 projects if they overestimate the projects' future profits or their own ability (Malmendier and Tate, 2005).

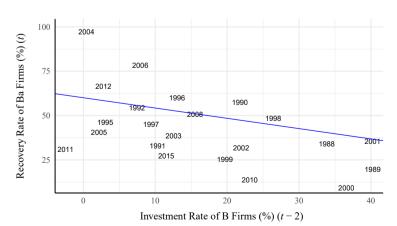
However, we argue that the over-investment hypothesis fails to predict the negative cross-rating correlation between investment and recovery rates. First, without any information asymmetry, risk-shifting only predicts a relationship within a firm, not across firms. Thus, this hypothesis can explain the coefficient of -0.288 on the Ba firms' investment rates on Ba firms' recovery rates but produces no predictions on the cross-rating relationship between investment and recovery rates. Second, even with some information asymmetry in the risk-shifting model, more investments in low-rated firms make them more distinguishable from high-rated firms that do not over-invest. As a result, high-rated firms should obtain a higher recovery value. In other words, the over-investment model should predict a positive cross-rating correlation between investment rate and recovery rate, which contradicts the prediction of our model and the empirical pattern in the data.

#### 6. Conclusion

Our model proposes a new informational channel through which a firm's investment decisions before default affect the information-driven liquidity in the secondary asset market. Because firms' investment decisions carry information about their asset quality, an increase in aggregate investment can reduce the value of information contained in firms' investment decisions, decreasing their recovery values in default and their debt and firm values.

Following recessions, various fiscal policy measures, e.g. the provision of bonus depreciation on qualifying capital investment, have been used to stimulate corporate investment. Ideally, a decrease in investment cost induces firms to make more profitable investments, boosting firm value and aggregate demand in the economy. However, our model suggests that such a policy can instead decrease firm value because it reduces the informational value of investment. Specifically, our model cautions against providing investment incentives to firms facing substantial default risks.

Finally, the informational value of investment for a firm's recovery value in bankruptcy provides a rational explanation for low recovery rates following investment waves where many firms invest. Through the lens of our model, when firms make waves of new investment due to positive economic outlooks, the amount of information contained in firms' investment decisions decreases. If a negative shock hits the economy following investment waves, firms



**Source(s):** Figure by authors

**Figure 7.** The figure above plots the relationship between the investment rate of B-rated firms at time t-2 and the recovery rate of Ba-rated firms at time t over the period from 1985 to 2014, consistent with column (2) in Table 2

that go bankrupt will suffer more from information asymmetry and obtain lower recovery rates. We leave the business cycle fluctuations of the informational value of investment for future research.

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#### Notes

- 1. In the United States Chapter 7 liquidation, potential buyers are under a time constraint to establish a stalking horse bid. A stalking horse bid is the first offer for the assets of a bankrupt firm which also sets the floor for the asset price, and the stalking-horse bidder typically carries several benefits, including expense reimbursements, breakup fees, and exclusivity for some period. Therefore, all potential bidders have an incentive to submit a bid quickly. Further, the overall Section 363 sales process typically carries a time limit for submitting bids of 20–30 days after the bankruptcy court approves the bid procedures. In a corporate restructuring by Chapter 11 re-organization, merger, or acquisition, potential buyers may include external investors who must evaluate targets from distressed firms without fully knowing their internal operations.
- 2. This interpretation appears reasonable based on the existing empirical literature studying corporate governance in financial distress. Eckbo and Thorburn (2003) study Swedish bankruptcy liquidation auctions and argues CEOs have private benefits of control that override risk-shifting incentives, so they handle the bankruptcy process on behalf of the debtholder in control to be rehired if the auction results in a going-concern sale. Khanna and Poulsen (1995) find corroborating evidence in the United States, finding that firms filing Chapter 11 bankruptcies did not appear to have statistically different investment behavior as other firms, even when controlling for managerial turnover or managerial ownership, and that when CEOs are replaced in bankruptcy, the market does not respond positively. Henderson (2007) study executive compensation for eighty bankrupt firms and find executive compensations largely resemble those from non-bankrupt firms, and Eckbo et al. (2016) show one-third of existing CEOs continue to serve in the executive suite after Chapter 11 bankruptcy.
- 3. See Senbet and Wang (2010) for a detailed literature review.
- 4. For instance, Titman and Tsyplakov (2007) and Diamond and He (2014) analyze the relation between debt overhang and debt maturity structure. He (2011) and Hackbarth et al. (2019) examine how the debt overhang problem interacts with managerial moral hazard and short-termism. Mello and Parsons (1992) and Moyen (2007) measure the agency cost of debt overhang and find the cost significant.
- 5. We can alternatively assume that the firm issues new equity to cover the operating losses.
- 6. The equilibrium outcome would not change even if we assume that the firm may fail to get external funding for new investment, say, in the Poisson manner, because such a one-time failure in investment can be ignored in the continuous-time setting.
- 7. In the real world, partial liquidation in bankruptcy is frequently observed. A notable example is General Motor's bankruptcy filing in 2009, during which the company sold off non-core assets such as the brands like Hummer and Saab while retaining its core assets. While various motives can drive partial liquidation, this paper particularly highlights the signaling role of partial liquidation.
- 8. Note that both investment and default thresholds are endogenous variables. But, in this section, we vary only the investment threshold to focus on the pure effect of the investment decision. We present the full comparative statics analysis in the next section.
- 9. As standard, the credit spread is defined as the bond yield minus the risk-free rate, where the bond yield y for a firm with fundamental x is determined from  $D(x) = \int_0^\infty e^{-yt} c dt$ .
- 10. This choice for data analysis trades off between the amount of information asymmetry and data availability. Firms are rarely rated Caa or below when they are not already in default or imminently close to default. Due to this limited data availability, the estimated investment rates of C-Caa firms may not be reliable. From our original Compustat data with 196,905 firm-year observations, firms with Caa ratings or below at least two years before default have only 1,259 firm-year observations.
- Out of 4,484 defaults from our Moody's data, only 100 observations belong to the firms rated A
  or above.

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## Appendix

#### A.1. First-Best Firm Value

The present value of an unlevered firm, denoted by  $V^{FB}(x)$ , satisfies the following Hamilton-Jacobi-Bellman equation:

$$rV^{FB}(x) = \max_{i \in [0,1]} x - i(gx + h) + (\mu + \delta i)xV_x^{FB}(x) + \frac{\sigma^2}{2}x^2V_{xx}^{FB}(x). \tag{11}$$

This maximization problem implies that the firm chooses to invest at time t if and only if  $\delta x_t V_x^{FB}(x_t) \geq g x_t + h$ . We conjecture that the firm invests when  $x_t \geq x_I^{FB}$  for some threshold  $x_I^{FB}$ . In the case of h=0, the firm always invests (that is,  $x_I^{FB}=0$ ), and the first-best firm value is equal to  $V^{FB}(x) = \frac{(1-g)x}{r-\mu-\delta}$ . This investment strategy is indeed optimal because condition (2) implies  $\delta V_x^{FB}(x) = \frac{\delta(1-g)}{r-\mu-\delta} > g$ ,  $\forall x>0$ . In the other case, in which h>0,  $V^{FB}(x)$  is equal to

$$V^{FB}(x) = \begin{cases} \frac{x}{r-\mu} + A_1 x^{\rho_1}, & \forall x \in (0, x_I^{FB}] \\ -\frac{h}{r} + \frac{(1-g)x}{r-\mu-\delta} + A_2 x^{\rho_2}, & \forall x \in [x_I^{FB}, \infty), \end{cases}$$

where

$$\begin{split} \rho_1 &= \frac{-\mu + \frac{\sigma^2}{2} + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2 r}}{\sigma^2} > 0, \\ \rho_2 &= \frac{-(\mu + \delta) + \frac{\sigma^2}{2} - \sqrt{\left(\mu + \delta - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2 r}}{\sigma^2} < 0. \end{split}$$

The coefficients,  $A_1$  and  $A_2$ , are determined from the following boundary conditions:

$$\lim_{x \uparrow x_t^{FB}} V^{FB}(x) = \lim_{x \downarrow x_t^{FB}} V^{FB}(x), \quad \lim_{x \uparrow x_t^{FB}} V^{FB}_x(x) = \lim_{x \downarrow x_t^{FB}} V^{FB}_x(x),$$

which leads to a system of linear equations. Further, the optimal threshold  $x_I^{FB}$  must satisfy the following indifference condition:  $\delta x_I^{FB} V_x^{FB} (x_I^{FB}) = g x_I^{FB} + h$ . Note that there exists at least one threshold satisfying this condition because h > 0, and when x is sufficiently large, we have  $\delta x V_x^{FB} (x) > g x + h$  due to condition (2). We omit to prove uniqueness and global optimality for this threshold-type investment strategy because we provide a similar argument in a more general setting when analyzing a levered firm; see Appendix A.4.

## A.2. Optimality of the Asset Retention Strategy

In this section, we prove the global optimality of the asset retention strategy,  $f(\cdot)$ . For any x and y such that  $x \neq y \in [x_D, x_I)$ , we have

$$f(x)\alpha x + (1 - f(x))\beta x - [f(y)\alpha x + (1 - f(y))\beta y]$$

$$= \int_{y}^{x} \frac{\partial}{\partial z} [f(z)\alpha x + (1 - f(z))\beta z] dz$$

$$= \int_{y}^{x} f'(z)\alpha (x - z) dz, \quad \text{by condition (8)},$$

$$\geq 0, \quad \text{because } f'(z) \geq 0.$$

Hence, the asset retention strategy satisfying condition (8) is globally optimal.

#### A.3. Debt Valuation

In this section, we compute the constants,  $\{A_1, \ldots, A_5, \zeta_1, \ldots, \zeta_5\}$ , that are used in the closed-form solution for the debt value.

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Case (1).  $x_I < x_L$ . In this case, the constants,  $\zeta_1, \ldots, \zeta_5$ , are given by

$$\begin{split} \zeta_1,\zeta_2 &= \frac{-\mu + \frac{\sigma^2}{2} \pm \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r+\phi)}}{\sigma^2},\\ \zeta_3,\zeta_4 &= \frac{-\mu - \delta + \frac{\sigma^2}{2} \pm \sqrt{\left(\mu + \delta - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r+\phi)}}{\sigma^2},\\ \zeta_5 &= \frac{-\mu - \delta + \frac{\sigma^2}{2} - \sqrt{\left(\mu + \delta - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2r}}{\sigma^2}. \end{split}$$

The coefficients,  $A_1, \ldots, A_5$ , are obtained from

$$D(x_D) = \frac{\beta x_D}{r - \mu}, \quad \lim_{x \uparrow x_I} D(x) = \lim_{x \downarrow x_I} D(x), \quad \lim_{x \uparrow x_I} D_x(x) = \lim_{x \downarrow x_I} D_x(x),$$
$$\lim_{x \uparrow x_L} D(x) = \lim_{x \downarrow x_L} D(x), \quad \lim_{x \uparrow x_L} D_x(x) = \lim_{x \downarrow x_L} D_x(x),$$

which leads to a system of linear equations.

Case (2).  $x_L \le x_I$ . In this case, the constants,  $\zeta_3$  and  $\zeta_5$  are given by

$$\zeta_3,\zeta_4=\frac{-\mu+\frac{\sigma^2}{2}\pm\sqrt{\left(\mu-\frac{\sigma^2}{2}\right)^2+2\sigma^2r}}{\sigma^2},$$

whereas  $\zeta_1, \zeta_2$ , and  $\zeta_5$  are the same as above. The coefficients,  $A_1, \ldots, A_5$ , are obtained from

$$D(x_D) = \frac{\beta x_D}{r - \mu}, \quad \lim_{x \uparrow x_L} D(x) = \lim_{x \downarrow x_L} D(x), \quad \lim_{x \uparrow x_L} D_x(x) = \lim_{x \downarrow x_L} D_x(x),$$
$$\lim_{x \uparrow x_I} D(x) = \lim_{x \downarrow x_I} D(x), \quad \lim_{x \uparrow x_I} D_x(x) = \lim_{x \downarrow x_I} D_x(x),$$

which leads to a system of linear equations.

# A.4. Equity Valuation

As the constants  $\{\zeta_1, \ldots, \zeta_5\}$  are given above, we only need to calculate  $\{B_1, \ldots, B_5\}$ . To this aim, we again consider two cases: (1)  $x_I < x_L$  and (2)  $x_L \le x_L$ .

Case (1).  $x_1 < x_L$ . In this case, the constants,  $B_1, \ldots, B_5$ , are obtained from

$$\begin{split} E(x_D) &= 0, \quad \underset{x \uparrow x_I}{\lim} E(x) = \underset{x \downarrow x_I}{\lim} E(x), \quad \underset{x \uparrow x_L}{\lim} E_x(x) = \underset{x \downarrow x_I}{\lim} E_x(x), \\ \underset{x \uparrow x_L}{\lim} E(x) &= \underset{x \downarrow x_L}{\lim} E_x(x), \quad \underset{x \uparrow x_L}{\lim} E_x(x) = \underset{x \downarrow x_L}{\lim} E_x(x), \end{split}$$

which leads to a system of linear equations.

Case (2).  $x_L \le x_I$ . In this case, the constants,  $B_1, \ldots, B_5$ , are obtained from

$$\begin{split} E(x_D) &= 0, \quad \underset{x \uparrow x_L}{\lim} E(x) = \underset{x \downarrow x_I}{\lim} E(x), \quad \underset{x \uparrow x_L}{\lim} E_x(x) = \underset{x \downarrow x_I}{\lim} E_x(x), \\ & \underset{x \uparrow x_I}{\lim} E(x) = \underset{x \downarrow x_I}{\lim} E_x(x), \quad \underset{x \uparrow x_I}{\lim} E_x(x) = \underset{x \downarrow x_I}{\lim} E_x(x), \end{split}$$

which leads to a system of linear equations.

# A.5. Data Description for Recovery Rates and Investment Rates

We use data from Compustat and Moody's Recovery Rate database. In this section, we detail the procedure and variables to calculate the investment rates and recovery rates, which involve aggregating credit ratings across firms and instruments through time.

From Compustat, we use data from the calendar year 1983 through 2016. The average net investment rate is the percentage change in the variable *PPENT*, which captures the net plants, property, and equipment. It is calculated at the firm level, and then we calculate the mean at the year-by-S&P credit rating level. To reduce the impact of outliers, we winsorize the investment rates at the 1 and 99% level and then take the mean. Compustat provides a firm's S&P credit rating, which we convert to the equivalent credit-rating scales used by Moody's, which ranges from {AAA, Aa, . . ., Ba, B, Caa-C}. When merging the data, we shift the investment rates back by two years and then merge with Moody's recovery-rate data consolidated at the credit rating-year level.

From Moody's, we have information on both credit ratings and default events. We use only the ratings on long-term senior unsecured bond ratings. First, we restrict the recovery rate data only to defaults that occurred at least one year after a firm receives a credit rating. Then, we also only use firms with credit ratings that are not stale. Allowing more stale ratings means we will have more observations, but the rating may be less informative. The recovery rate data are based on disaggregated firm-debt instrument-level data. For each firm, we may observe multiple debt instruments at the same point in time. Moreover, depending on the seniority level and where the debt instrument sits in the firm's capital structure, the same firm may have different credit ratings across different instruments. One default event may correspond to multiple debt-level recovery rates. Yet, we do not want to lose the information content in the ratings across different instruments within the same firm.

To consolidate the recovery-rate data, we take a debt-outstanding-weighted average at the firm-default event-credit-rating level. For example, if a firm simultaneously defaults on three instruments rated Ba, Ba, and B, with outstanding values 100, 50, and 120, respectively, Moody's records as this one default event. For this default event, we calculate the firm's Ba-debt recovery rate based on a weight of two-thirds and one-third for the first and second Ba-rated debt instruments, respectively. Then, we aggregate the recovery rates by taking the simple average across all credit ratings by year. To reduce the impact of outliers, we winsorize the recovery rates at the 1 and 99% level and then take the mean.

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