# Size Effects on the Scattering of Electron and Spherical Dust Grain in Dusty Plasmas

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The finite size effects of the charged dust grain on the electron-dust grain collisions are investigated in complex dusty plasmas. The stationary phase analysis and the effective potential due to the renormalized dust charge are employed to obtain the phase shift for the scattering of the electron and the spherically charged dust grain as a function of the impact parameter, collision energy, Debye length, and dust radius. It is found that the size effect of the dust grain enhances the electron-dust grain scattering cross section in dusty plasmas. It is also found that the size effect on the scattering cross section increases with increasing plasma density. In addition, it is found that the size effect on the electron-dust scattering cross section decreases with an increase of the plasma temperature.

*Key words:* Size Effects; Electron-Dust Grain Scattering; Dusty Plasmas.

Recently, there has been a considerable interest in the dynamics of complex plasmas consisting of electrons, ions, and solid macroparticles encompassing collective interactions and nonlinear effects [1 – 5]. It has been found that these complex plasmas are ubiquitous not only in space plasmas but also in various laboratory plasmas. Hence, various physical and chemical processes of charged dust grains have been extensively investigated in order to understand the plasma properties of the complex dusty plasmas [3, 6]. The standard Debye-Hückel interaction potential has been widely used to investigate the collision and radiation processes in weakly coupled plasmas since the average interaction energy between charged particles is found to be smaller than the average kinetic energy of a plasma particle [7]. However, the electrostatic interaction including the spherical micropaticles in complex dusty plasmas would be different from the conventional screened Yukawa type potential due to the charge renormalization on the surface of the dust grain and the effect of the finite dust size. Very recently, an excellent work [8] on the effective potential and the renormalized charge of the dust grain are given for the electrostatic interaction of spherical microparticles in dusty plasmas. Hence, it would be expected that the scattering of spherically charged dust grains would be different from that of point charges due to the finite size of the dust grain. In addition, the understanding of the size effects on the dust grain collisions would have fundamental and practical importance in physics and modelling of complex dusty plasmas. Thus, in this paper we investigate the finite size effects of the dust grain on the electron-dust grain collisions since the scattering process provides useful information on various plasma parameters. It has been shown that the stationary phase method [9] is quite useful in the collision processes since the scattering cross section can be expressed by a closed integral form with the scattering phase shift including the information on the plasma parameters and the dynamics of the collision system. Hence, in this work the stationary phase method and the effective interaction potential including the renormalized dust charge are engaged in order to investigate the finite size effect on the scattering cross section for the electron-dust grain collision in dusty plasmas as a function of the impact parameter, collision energy, Debye length, and dust radius.

Using the method of the stationary phase [9], the partial-wave expression of the total elastic scattering cross section $\sigma_\mathrm{T}$ would be represented by the integral over the impact parameter $b$:

$$\begin{aligned}\sigma_\mathrm{T} &= 4\pi\lambda\!\!\!^{-2}\sum_{l=0}^{\infty}(2l+1)\sin^2\eta_l \\ &= 8\pi\int \mathrm{d}b\, b\sin^2\delta(b),\end{aligned} \tag{1}$$

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where $\lambdabar$ is the reduced de Broglie wave length of the collision system and $\eta_l$ is the partial-wave phase shift for the angular momentum $l$. Here, $\delta(b)$ stands for the scattering phase for the interaction potential $V(r)$ defined as

$$\begin{aligned}\delta(b) &= -\frac{1}{2\hbar}\int_{-\infty}^{\infty} \mathrm{d}t V(r) \\ &\cong -\frac{1}{\hbar v}\int_{0}^{\infty} \mathrm{d}z \exp\left[\ln V(b) + \frac{z^2}{2}\frac{\mathrm{d}V(b)/\mathrm{d}b}{bV(b)}\right],\end{aligned} \tag{2}$$

where $r = (b^2+z^2)^{1/2}$, $z \equiv vt$, $\hbar$ is the rationalized Planck constant, and $v$ is the collision velocity. The total elastic scattering cross section [9] would then be obtained in the following form:

$$\sigma_{\mathrm{T}} = 8\pi \int \mathrm{d}bb \sin^2\left[\frac{V(b)}{\hbar v}\left(-\frac{\pi}{2}\frac{bV(b)}{\mathrm{d}V(b)/\mathrm{d}b}\right)^{1/2}\right]. \tag{3}$$

Very recently, the extremely useful analytic expressions [8] of the effective electrostatic interaction potential between spherical microparticles and the renormalized charge of the dust grain have been obtained by the Legendre polynomial decomposition of microparticle potential distributions in dusty plasmas. According to the effective or renormalized dust charge [8], $q_{1\mathrm{eff}}$ $[= q_1 \mathrm{e}^{k_{\mathrm{D}}a_1}/(1+k_{\mathrm{D}}a_1)]$ and the finite size effect of the dust grain, the effective interaction potential $V_{\mathrm{eff}}(R)$ between the point particle with charge $q_2 e$ and the spherically charged dust grain with charge $q_1 e$ in the range $R \ll \lambda_{\mathrm{D}}$ would be given by

$$\begin{aligned}V_{\mathrm{eff}}(R) = &\frac{q_1 q_2 e^2}{R}\exp(-k_{\mathrm{D}}R)\left(1+\frac{1}{2}k_{\mathrm{D}}^2 a_1^2\right) \\ &-\frac{q_2^2 e^2}{2R^2}\frac{a_1^3}{R^2-a_1^2},\end{aligned} \tag{4}$$

where $k_{\mathrm{D}}$ $(= 1/\lambda_{\mathrm{D}})$ is the inverse Debye length, $\lambda_{\mathrm{D}}$ is the Debye length, $a_1$ is the radius of the dust grain, and $R$ is the interparticle distance. If we neglect the size of the charged dust grain, however, the interaction potential would then be the standard Debye-Hückel form $V_{\mathrm{DH}}(R) = (q_1 q_2 e^2/R)\exp(-k_{\mathrm{D}}R)$. Very recently, an excellent investigation [10] on numerous aspects of collective interactions in dusty plasmas was given by Shukla and Eliasson. After some mathematical manipulations using the effective potential, the total elastic scattering cross section $\bar{\sigma}_{\mathrm{T}}$ in units of $\pi a_0^2$ for the collision of the electron by the spherically charged dust grain with charge $qe$ in complex dusty plasmas including the finite size effects would be obtained in the following form:

$$\bar{\sigma}_{\mathrm{T}} = 8\int_{2\bar{a}}^{\bar{\lambda}_{\mathrm{D}}} \mathrm{d}\bar{R}\bar{R}\sin^2\left[J(\bar{R},\bar{k}_{\mathrm{D}},\bar{a})\right], \tag{5}$$

where $a_0$ $(= \hbar^2/me^2)$ is the Bohr radius, $\bar{\lambda}_{\mathrm{D}}$ $(\equiv \lambda_{\mathrm{D}}/a_0)$ is the scaled Debye length, $\bar{a} \equiv a/a_0$, $\bar{R} \equiv R/a_0$, $\bar{k}_{\mathrm{D}} \equiv k_{\mathrm{D}}/a_0$, and the argument function $J(\bar{R},\bar{k}_{\mathrm{D}},\bar{a})$ including the finite size effects is found to be

$$J(\bar{R},\bar{k}_{\mathrm{D}},\bar{a}) = \left(\frac{\pi}{2\bar{E}}\right)^{1/2}\left[\frac{\left(q\exp(-\bar{k}_{\mathrm{D}}\bar{R})(1+\bar{k}_{\mathrm{D}}^2\bar{a}^2/2)+\frac{\bar{a}^3}{2\bar{R}(\bar{R}^2-\bar{a}^2)}\right)^3}{q\exp(-\bar{k}_{\mathrm{D}}\bar{R})(1+\bar{k}_{\mathrm{D}}\bar{R})(1+\bar{k}_{\mathrm{D}}^2\bar{a}^2/2)-\frac{\bar{a}_1^3(2\bar{R}^2-\bar{a}^2)}{\bar{R}(\bar{R}^2-\bar{a}^2)^3}}\right]^{1/2}, \tag{6}$$

where $\bar{E}$ $(\equiv \mu v^2/2Ry)$ is the scaled collision energy, $\mu$ is the reduced mass of the collision system, and $Ry$ $(= me^4/2\hbar^2 \approx 13.6\ \mathrm{eV})$ is the Rydberg constant. These size dependences of the integrand in (5) would provide useful information on the collision dynamics involving the spherically charged dust grains. Since the absorption phenomena would be more important than the collision phenomena in the domain $a < R < 2a$, the lower limit of the integration in (5) would be given by $2\bar{a}$. However, if we neglect the size effects on the electron-dust grain interaction, the total electron-dust grain scattering cross section $\bar{\sigma}'_{\mathrm{T}}$ in units of $\pi a_0^2$ becomes

$$\bar{\sigma}'_{\mathrm{T}} = 8\int_{2\bar{a}}^{\bar{\lambda}_{\mathrm{D}}} \mathrm{d}\bar{R}\bar{R}\sin^2[K(\bar{R},\bar{k}_{\mathrm{D}})], \tag{7}$$

where the function $K(\bar{R},\bar{k}_{\mathrm{D}})$ is given by

$$K(\bar{R},\bar{k}_{\mathrm{D}}) = \left(\frac{\pi}{2\bar{E}}\right)^{1/2}\left[\frac{q^2\exp(-2\bar{k}_{\mathrm{D}}\bar{R})}{(1+\bar{k}_{\mathrm{D}}\bar{R})}\right]^{1/2}. \tag{8}$$

Then, the function $F(\bar{\lambda}_{\mathrm{D}},\bar{a})$ of the finite size effect on

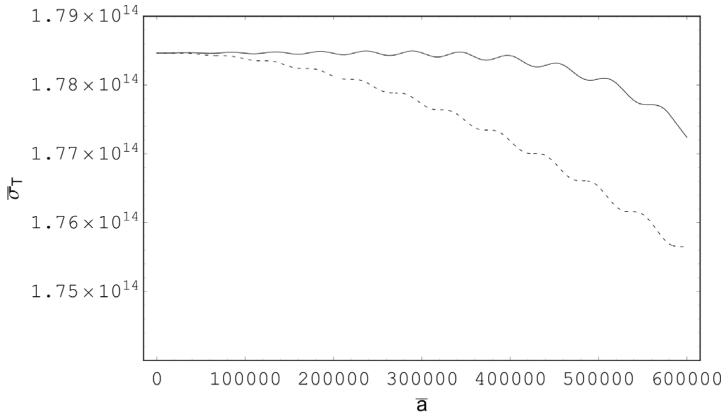


Fig. 1. Total scaled scattering cross section ($\bar{\sigma}_{\rm T}$) as a function of the scaled dust radius ($\bar{a}$) for $\bar{k}_{\rm D} = 1.06\times10^{-7}$ and $\bar{E}=1$. The solid line represents the scattering cross section including the finite size effects. The dotted line represents the scattering cross section neglecting the finite size effects.

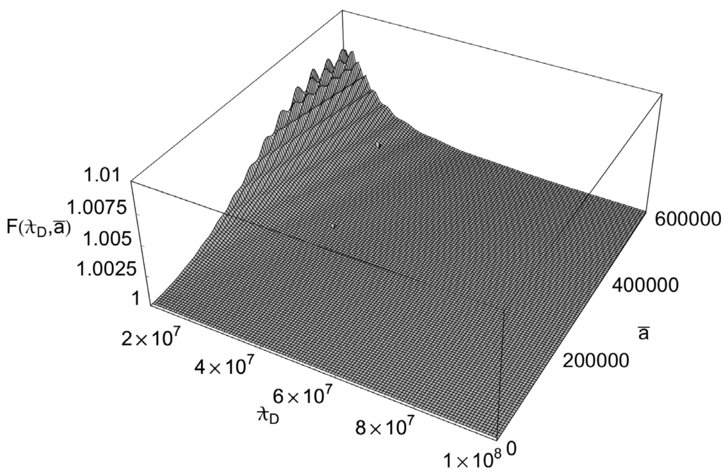


Fig. 2. Three-dimensional plot of the function $F$ of the size effect on the electron-dust grain scattering cross section as a function of the scaled Debye length ($\bar{\lambda}_{\rm D}$) and the scaled dust radius ($\bar{a}$) for $\bar{E}=1$.

the electron-dust grain scattering cross section would be obtained as follows:

$$F(\bar{\lambda}_{\rm D},\bar{a}) = \frac{\int_{2\bar{a}}^{\bar{\lambda}_{\rm D}} {\rm d}\bar{R}\bar{R}\sin^2\left[\sqrt{\frac{\pi}{2\bar{E}}}\left(\frac{\left(qe^{-\bar{k}_{\rm D}\bar{R}}(1+\bar{k}_{\rm D}^2\bar{a}^2/2)+\frac{\bar{a}^3}{2\bar{R}(\bar{R}^2-\bar{a}^2)}\right)^3}{qe^{-\bar{k}_{\rm D}\bar{R}}(1+\bar{k}_{\rm D}\bar{R})(1+\bar{k}_{\rm D}^2\bar{a}^2/2)-\frac{\bar{a}_1^3(2\bar{R}^2-\bar{a}^2)}{\bar{R}(\bar{R}^2-\bar{a}^2)^3}}\right)^{1/2}\right]}{\int_{2\bar{a}}^{\bar{\lambda}_{\rm D}} {\rm d}\bar{R}\bar{R}\sin^2\left[\sqrt{\frac{\pi}{2\bar{E}}}\left(\frac{q^2\exp(-2\bar{k}_{\rm D}\bar{R})}{(1+\bar{k}_{\rm D}\bar{R})}\right)^{1/2}\right]}. \quad (9)$$

In order to specifically investigate the finite size effects on the collisions involving the spherically charged dust grain, we set $\bar{a} < \bar{\lambda}_{\rm D}$, $q = 500$, and the mass density of the dust grain is $\rho_{\rm m} \cong 2{\rm g/cm}^3$ since the Debye length is generally much greater than the dust radius in typical environments of complex dusty plasmas [3]. Figure 1 represents the total scaled electron-dust grain scattering cross section ($\bar{\sigma}_{\rm T}$) as a function of the scaled dust radius ($\bar{a}$). As shown, the electron-dust scattering cross sections including the finite size effects are found to be greater than those neglecting the size effects. Hence, we found that the finite size effect of the dust grain enhances the electron-dust grain scattering cross section in dusty plasmas. Figure 2 shows the three-dimensional plot of the function $F(\bar{\lambda}_{\rm D},\bar{a})$ of the size effect on the electron-dust grain scattering cross section as a function of the scaled Debye length and the scaled dust radius. As it is seen, it is found that the size effect on the electron-dust scattering cross section increases with decreasing Debye length. Hence, it is interesting to note that the finite size effect on the scattering cross section increases with increasing density of the dusty plasma. It can be also expected that the size effect on the electron-dust scattering cross section decreases with an increase of the plasma temperature. In this work, we have found that the finite size effect of the spherically charged dust grain plays an important role in the electron-dust collisions in dusty plasmas. Therefore, the finite size effects should be appropriately taken into account to obtain the correct information on the electron-dust collision dynamics in dusty plasmas. These results would provide useful information on the finite size effects on the collision dynamics involving the charged spherical microparticles in complex dusty plasmas.

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