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Research article

Generalized (α,β,γ)-derivations on Lie C^*-algebras

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Abstract: The Hyers-Ulam stability of (α,β,γ)-derivations on Lie C^*-algebras is discussed by following functional inequality

$$f(ax+by)+f(ax-by)=2f(ax)+bf(y)+bf(-y),$$

where a, b are nonzero fixed complex numbers.

Keywords: Hyers-Ulam stability; (α,β,γ)-derivation; Lie C^*-algebra
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1. Introduction and preliminaries

The derivation theory of Lie algebras play a key role in Lie theory. In particular, Physically motivated relations between two Lie algebras have been extensively discussed [27]. The problems for the structures and characteristics of (α,β,γ)-derivations of Lie algebras have been extensively investigated by a range of scholars, as for this, many scholars have made useful researches (see [22, 28, 37]). The authors set up the structure and properties of (α,β,γ)-derivations of Lie algebras.

In this work, The definition of a Lie C^*-algebra come from [29, 30, 34]). In [28], the definition of (α,β,γ)-derivation can be found.

1940, the stability problem of group homomorphisms was raised by Ulam [38]. In 1941, Hyers [20] answers this question with a qualified yes to the question of Ulam for additive groups in Banach spaces. Hyers' theorem was generalized by Aoki [2], Rassias [35] and Găvruta [17] for linear mappings. In recent years, a lot of experts and scholars have studied in this area and made many achievements (see [1, 3, 6, 7, 9, 12, 23–25, 33, 39, 40]).

Gilányi [18] and [36] considered the functional inequality

$$\|2f(x) + 2f(y) - f(x - y)\| \leq \|f(x + y)\| \tag{1.1}$$

then f satisfies the Jordan-von Neumann functional equation

$$2f(x) + 2f(y) = f(x + y) + f(x - y),$$

respectively. The Hyers-Ulam stability of the above functional inequality is discussed by Fechner [16] and Gilányi [19]. Park [31, 32] gave the definition of additive ρ-functional inequalities and discussed the Hyers-Ulam stability of the additive ρ-functional inequalities in different spaces .

To obtain a Jordan and von Neumann type characterization theorem for the quasi-inner-product spaces, Drygas [11] considered the functional equation

$$f(x + y) + f(x - y) = 2f(x) + f(y) + f(-y), \tag{1.2}$$

which solution is called a *Drygas mapping*. The general solution of the above functional equation was given by Ebanks, Kannappan and Sahoo [13] as

$$f(x) = Q(x) + A(x),$$

here A is an additive mapping and Q is a quadratic mapping.

In this work, we consider the stability of (α, β, γ)-derivations on Lie C^*-algebras by the general Drygas functional equation

$$f(ax + by) + f(ax - by) = f(2ax) + bf(y) + bf(-y), \tag{1.3}$$

the coefficients a, b is complex number, the proof of stability of the (1.3) is difference in [13]. The additive mapping A and quadratic mapping Q is constructed by the function relations, this method is called "directed method". In the (1.3), a, b action will cause difficulties for the stability of functional inequalities. We can overcome the influence of a, b, the stability of (α, β, γ)-derivations using the fixed method. The beautiful examples about (α, β, γ)-derivations can be found in [41].

The Hyers-Ulam stability analysis on C^*-algebras about functional equations have been discussed by fixed point theorem (see [5, 8, 14, 15, 21]).

Next, the concept of the "generalized complete metric space" is introduced following Luxemburg [26].

Definition 1.1. Let X be an abstract (nonempty) set, the elements of which are denoted by $x, y, \cdots$ and assume that on the Cartesian product $X \times X$ a distance function $d(x, y)(0 \leq d(x, y) \leq \infty)$ is defined, satisfying the following conditions

(1) $d(x, y) = 0$ if and only if $x = y$,
(2) $d(x, y) = d(y, x)$(symmetry),
(3) $d(x, y) \leq d(x, z) + d(z, y)$(triangle inequality),
(4) every d-Cauchy sequence in X is d-convergent, i.e. $\lim_{n,m\to\infty} d(x_n, x_m) = 0$ for a sequence $x_n \in X(n = 1, 2, \cdots)$ implies the existence of an element $x \in X$ with $\lim_{n\to\infty} d(x, x_n) = 0$, ($x$ is unique).

By the concept, every two points in X may be have the infinite distance. The space is called a *generalized complete metric space*.

We recall fixed point theorem that plays an key role to prove the stability of derivation.

Theorem 1.2. *[4, 10] Let (X, d) be a complete generalized metric space and $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for any $x \in X$, either*

$$d(J^n x, J^{n+1} x) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) $d(J^n x, J^{n+1} x) < \infty$, for all $n \geq n_0$;
(2) the sequence $\{J^n x\}$ converges to a fixed point y^* of J;
(3) y^* is the unique fixed point of J in the set $Y = \{y \in X | d(J^{n_0} x, y) < \infty\}$;
(4) $d(y, y^*) \leq \frac{1}{1-L} d(y, Jy)$ for all $y \in Y$.

Now, using some thoughts from ([4, 10, 15]) we discuss the stability for (α, β, γ)-derivations and Lie C^*-algebra homomorphisms on Lie C^*-algebras related to (1.3) via the above fixed point theorem.

2. The stability of (α, β, γ)-derivations

Now, suppose that s is complex fixed point and $\mathcal{A}$ is a Lie C^*-algebra with norm $\|\cdot\|$. The following lemma is necessary to prove our main theorems.

Lemma 2.1. *[30] Suppose X and Y are linear spaces, $f : X \to Y$ is an additive map satisfying $f(\mu x) = \mu f(x)$, $\forall x \in X$ and $\mu \in T^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$. Then f is $\mathbb{C}$-linear.*

Lemma 2.2. *Assume $f : \mathcal{A} \to \mathcal{A}$ is a map satisfying*

$$\begin{aligned} &\|f(ax+by) + f(ax-by) - f(2ax) - bf(y) - bf(-y)\| \\ &\leq \|s(f(ax-by) + f(ax+by) - f(2ax))\| \end{aligned} \tag{2.1}$$

$\forall x, y \in \mathcal{A}$, $|s| \leq |1 - 2b| \leq 1$. *Then f is additive.*

Proof. If $x = y = 0$ in (2.1), then $f(0) = 0$. If $x = \frac{b}{a} y$ in (2.1) with $b \neq 0$, one obtain $f(-y) = -f(y)$.

Next, we discuss that f is additive. Since $f(-y) = -f(y)$ in (2.1),

$$f(ax + by) + f(ax - by) - f(2ax) = 0$$

for $\forall x, y \in \mathcal{A}$. So f is additive. □

Theorem 2.3. *If there are a mapping $\phi : \mathcal{A}^2 \to [0, \infty)$*

$$\frac{1}{2}\phi(2x, 2y) \leq L\phi(x, y), \quad \forall x, y \in \mathcal{A}; \tag{2.2}$$

and a mapping $\psi : \mathcal{A}^2 \to [0, \infty)$ with a constant $0 < L < 1$

$$\psi\left(\frac{x}{2}, \frac{y}{2}\right) \leq L^2 \frac{1}{2^2} \psi(x, y), \quad \forall x, y \in \mathcal{A}. \tag{2.3}$$

Let $f:\mathcal{A}\to\mathcal{A}$ satisfy

$$\begin{aligned}&\|f(a\mu x+by)+f(a\mu x-by)-\mu f(2ax)-bf(y)-bf(-y)\|\\&\le\|s(f(a\mu x-by)+f(a\mu x+by)-\mu f(2ax))\|+\phi(x,y),\end{aligned}\tag{2.4}$$

$$\|\alpha f[x,y]-\beta[f(x),y]-\gamma[x,f(y)]\|\le\psi(x,y),\tag{2.5}$$

$\forall x,y\in\mathcal{A},\mu\in T^1$, *some* α,β,γ,a,b *and* $|s|\le|1-2b|\le1$. *Then we can find a unique* (α,β,γ)*-derivation* $\delta:\mathcal{A}\to\mathcal{A}$ *satisfies (1.3) and*

$$\|f(x)-\delta(x)\|\le\frac{1}{2(1-|s|)(1-L)}\phi\left(\frac{x}{a},0\right),\quad\forall x\in\mathcal{A}.\tag{2.6}$$

Proof. Suppose Ω is a set of all mappings from $\mathcal{A}$ into $\mathcal{A}$, on Ω, a generalized metric is introduced,

$$d(g,h)=\inf\left\{C\in\mathbb{R}^+:\|g(x)-h(x)\|\le C\phi\left(\frac{x}{a},0\right),\forall x\in\mathcal{A}\right\}.$$

Then (Ω,d) becomes a generalized complete metric space. One define a map $T:\Omega\to\Omega$ by

$$Tg(x)=\frac{1}{2}g(2x),\forall g\in\Omega,x\in\mathcal{A}.$$

Let $g,h\in\Omega$ with $d(g,h)\le C$, here $C\in(0,\infty)$ is an arbitrary constant. Then we obtain $\|g(x)-h(x)\|\le C\phi\left(\frac{x}{a},0\right)$,

$$\|Tg(x)-Th(x)\|\le\frac{C}{2}\phi(2x,0)\le LC\phi(x,0),\forall x\in\mathcal{A},$$

i.e. $d(Tg-Th)\le Ld(g,h),\forall g,h\in\Omega$. Therefore, T is a strictly contractive self-mapping on Ω associated with the Lipschitz constant L.

If $x=y=0$ in (2.4), $f(0)=0$.

If $y=0$ and $\mu=1$ in (2.4), then

$$\|2f(ax)-f(2ax)\|\le|s|\|2f(ax)-f(2ax)\|+\phi(x,0),\quad\forall x\in\mathcal{A}.$$

Thus

$$\left\|\frac{f(2x)}{2}-f(x)\right\|\le\frac{1}{1-|s|}\frac{1}{2}\phi\left(\frac{x}{a},0\right)$$

for $\forall x\in\mathcal{A}$. Then we have $d(Tf,f)\le\frac{1}{2(1-|s|)}$. By Theorem 1.2, there is a unique fixed point of T, map δ, in the set $\Omega_1=\{g\in\Omega:d(f,g)<\infty\}$,

$$\delta(x):=\lim_{n\to\infty}\frac{1}{2^n}f(2^nx),\forall x\in\mathcal{A},\tag{2.7}$$

since $\lim_{n\to\infty}d(T^nf,\delta)=0$. Again by Theorem 1.2,

$$d(f,\delta)\le\frac{1}{1-L}d(Tf,f)\le\frac{1}{2(1-|s|)(1-L)},\forall x\in\mathcal{A}.$$

Then (2.6) holds.

By (2.4) and (2.7) and the property of ϕ,

$$\begin{aligned}
&\|\delta(a\mu x + by) + \delta(a\mu x - by) - \mu\delta(2ax) - b\delta(y) - b\delta(-y)\| \\
&= \lim_{n\to\infty} \frac{1}{2^n} \|f(2^n a\mu x + 2^n by) + f(2^n a\mu x - b2^n y) - \mu f(2a2^n x) \\
&-bf(2^n y) - bf(-2^n y)\| \\
&\leq \lim_{n\to\infty} \frac{1}{2^n} \|s(f(a\mu 2^n x + b2^n y) + f(a\mu 2^n x - b2^n y) - \mu f(2a2^n x))\| \\
&+ \lim_{n\to\infty} \frac{1}{2^n}\phi(2^n x, 0) \\
&\leq \|s(\delta(\mu ax + by) + \delta(a\mu x - by) - \mu\delta(2ax))\| + \lim_{n\to\infty} L^n\phi(x, 0).
\end{aligned}$$

That is, δ is additive by Lemma 2.2. Next, letting $y = 0$, we get $2\delta(a\mu x) = \mu\delta(2ax)$ and so the map δ is $\mathbb{C}$-linear. Therefore, by the property of ψ, (2.5) and (2.7), then

$$\begin{aligned}
&\|\alpha\delta[x, y] - \beta[\delta(x), y] - \gamma[x, \delta(y)]\| \\
&= \lim_{n\to\infty} 4^n \|\alpha f(\frac{[x, y]}{2^n \cdot 2^n}) - \beta[f(x/2^n), y/2^n] - \gamma[x/2^n, f(y/2^n)]\| \\
&\leq \lim_{n\to\infty} 4^n \psi\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \\
&\leq \lim_{n\to\infty} L^{2n}\psi(x, y) = 0
\end{aligned}$$

for $\forall x, y \in \mathcal{A}$, some α, β and $\gamma \in \mathbb{C}$. Thus

$$\alpha\delta[x, y] = \beta[\delta(x), y] + \gamma[x, \delta(y)], \forall x, y \in \mathcal{A},$$

for some α, β and $\gamma \in \mathbb{C}$. Hence δ is an unique derivation satisfying (2.6). □

Corollary 2.4. *If r, k and θ belong to real numbers, $0 < r < 1, 0 < k < 2$ and $\theta \geq 0$. Let the map $f : \mathcal{A} \to \mathcal{A}$ satisfy*

$$\begin{aligned}
&\|f(a\mu x + by) + f(a\mu x - by) - \mu f(2ax) - bf(y) - bf(-y)\| \\
&\leq \|s(f(a\mu x - by) + f(a\mu x + by) - \mu f(2ax))\| + \theta(\|x\|^r + \|y\|^r),
\end{aligned}$$

$$\|\alpha f[x, y] - \beta[f(x), y] - \gamma[x, f(y)]\| \leq \theta(\|x\|^k + \|y\|^k)$$

for $\forall x, y \in \mathcal{A}, \mu \in T^1$ and $|s| \leq |1 - 2b| \leq 1$. Then we can find a unique (α, β, γ)-derivation $\delta : \mathcal{A} \to \mathcal{A}$,

$$\|f(x) - \delta(x)\| \leq \frac{1}{(1 - |s|)|a|^r(2 - 2^r)}\|x\|^r$$

for $\forall x \in \mathcal{A}$.

Proof. Let $\phi(x, y) = \theta(\|x\|^r + \|y\|^r), \psi(x, y) = \theta(\|x\|^k + \|y\|^k)$ and $L = 2^{r-1}$ in Theorem 2.3, the desired result is obtained. □

Theorem 2.5. *If there exists a map $\psi : \mathcal{A}^2 \to [0,\infty)$ satisfying (2.3). Let a map $f : \mathcal{A} \to \mathcal{A}$ satisfy*

$$\begin{aligned} &\|f(a\mu x + by) + f(a\mu x - by) - \mu f(2ax) - bf(y) - bf(-y)\| \\ &\leq \|s(f(a\mu x - by) + f(a\mu x + by) - \mu f(2ax))\|, \end{aligned} \tag{2.8}$$

$$\|\alpha f[x,y] - \beta[f(x), y] - \gamma[x, f(y)]\| \leq \psi(x,y) \tag{2.9}$$

for $\forall x, y \in \mathcal{A}, \mu \in T^1$ and $|s| \leq |1-2b| \leq 1$. Thus the map $f : \mathcal{A} \to \mathcal{A}$ is a (α,β,γ)-derivation.

Proof. Let $\mu = 1$ in (2.8), the map f is additive by Lemma 2.2. Let $y = 0$ in (2.8), we get

$$\|2f(a\mu x) - \mu f(2ax)\| \leq 0$$

for $\forall x \in \mathcal{A}$, $\mu \in T^1$. So $f(\mu x) = \mu f(x), \forall x \in \mathcal{A}$ and $\mu \in T^1$. The map f is $\mathbb{C}$-linear by Lemma 2.1. On account of f is additive, by (2.9),

$$\begin{aligned} &\|\alpha f([x,y]) - \beta[f(x), y] - \gamma[x, f(y)]\| \\ &= \lim_{n\to\infty} 4^n \|\alpha f\left(\frac{[x,y]}{2^n \cdot 2^n}\right) - \beta\left[f\left(\frac{x}{2^n}\right), \frac{y}{2^n}\right] - \gamma\left[\frac{x}{2^n}, f\left(\frac{y}{2^n}\right)\right]\| \\ &\leq \lim_{n\to\infty} L^{2n}\psi(x,y) = 0 \end{aligned}$$

for $\forall x, y \in \mathcal{A}$. Thus

$$\alpha f([x,y]) = \beta[f(x), y] + \gamma[x, f(y)], \forall x, y \in \mathcal{A}.$$

□

Corollary 2.6. *If k and θ belong to real numbers with $0 < k < 2$ and $\theta \geq 0$. Assume a map $f : \mathcal{A} \to \mathcal{A}$ satisfies*

$$\begin{aligned} &\|f(a\mu x + by) + f(a\mu x - by) - \mu f(2ax) - bf(y) - bf(-y)\| \\ &\leq \|s(f(a\mu x - by) + f(a\mu x + by) - \mu f(2ax))\|, \end{aligned}$$

$$\|\alpha f[x,y] - \beta[f(x), y] - \gamma[x, f(y)]\| \leq \theta(\|x\|^k + \|y\|^k)$$

for $\forall x, y \in \mathcal{A}, \mu \in T^1$ and $|s| \leq |1-2b| \leq 1$. Then the map f is a (α,β,γ)-derivation.

Lemma 2.7. *If $f : \mathcal{A} \to \mathcal{A}$ is a map satisfying*

$$\begin{aligned} &\|f(ax + by) + f(ax - by) - f(2ax) - bf(y) - bf(-y)\| \\ &\geq \|s(f(ax - by) + f(ax + by) - f(2ax))\| \end{aligned}$$

for $\forall x, y \in \mathcal{A}$, $|s| \geq |1-2b| \geq 1$. Then f is additive.

Proof. Using the same technique with the Lemma 2.2, we can show that the Lemma 2.7. □

Theorem 2.8. *Assume the map* $\phi : \mathcal{A}^2 \to [0,\infty)$ *satisfies (2.2) and a map* $\psi : \mathcal{A}^2 \to [0,\infty)$ *satisfies (2.3). Let the map* $f : \mathcal{A} \to \mathcal{A}$ *satisfy*

$$\begin{aligned}&\|f(a\mu x+by)+f(a\mu x-by)-\mu f(2ax)-bf(y)-bf(-y)\|\\&\geq \|s(f(a\mu x-by)+f(a\mu x+by)-\mu f(2ax))\|-\phi(x,y),\end{aligned}$$

$$\|\alpha f[x,y]-\beta[f(x),y]-\gamma[x,f(y)]\|\leq \psi(x,y)$$

for $\forall x,y\in\mathcal{A},\mu\in T^1$*, some* α,β,γ,a,b*, and* $|s|\geq|1-2b|\geq 1$. *Then we can find a unique derivation* δ *satisfying (1.3), and*

$$\|f(x)-\delta(x)\|\leq \frac{1}{2(1-|s|)(1-L)}\phi\left(\frac{x}{a},0\right)$$

for $\forall x\in\mathcal{A}$.

Proof. In a similar vein of Theorem 2.3, the theorem can be proved. □

Corollary 2.9. *Suppose* $r,k,\theta\in\mathbb{R}$ *and* $0<r<1, 0<k<2$, $\theta\geq 0$*, let the map* $f:\mathcal{A}\to\mathcal{A}$ *satisfy*

$$\begin{aligned}&\|f(a\mu x+by)+f(a\mu x-by)-\mu f(2ax)-bf(y)-bf(-y)\|\\&\geq \|s(f(a\mu x-by)+f(a\mu x+by)-\mu f(2ax))\|-\theta(\|x\|^r+\|y\|^r),\end{aligned}$$

$$\|\alpha f[x,y]-\beta[f(x),y]-\gamma[x,f(y)]\|\leq \theta(\|x\|^k+\|y\|^k)$$

for $\forall x,y\in\mathcal{A},\mu\in T^1$*, some* α,β,γ,a,b*, and* $|s|\geq|1-2b|\geq 1$. *Then there is only one* (α,β,γ)*-derivation* $\delta:\mathcal{A}\to\mathcal{A}$ *satisfying*

$$\|f(x)-\delta(x)\|\leq \frac{1}{(1-|s|)|a|^r(2-2^r)}\|x\|^r$$

for $\forall x\in\mathcal{A}$.

Proof. In Theorem 2.8, let $\phi(x,y)=\theta(\|x\|^r+\|y\|^r),\psi(x,y)=\theta(\|x\|^k+\|y\|^k)$, $\forall x,y\in\mathcal{A}$ and $L=2^{r-1}$, then the Corollary is proved. □

Theorem 2.10. *If the map* $\psi:\mathcal{A}^2\to[0,\infty)$ *satisfies (2.3). The map* $f:\mathcal{A}\to\mathcal{A}$ *satisfies*

$$\begin{aligned}&\|f(a\mu x+by)+f(a\mu x-by)-\mu f(2ax)-bf(y)-bf(-y)\|\\&\geq \|s(f(a\mu x-by)+f(a\mu x+by)-\mu f(2ax))\|,\end{aligned}$$

$$\|\alpha f[x,y]-\beta[f(x),y]-\gamma[x,f(y)]\|\leq \psi(x,y)$$

for $\forall x,y\in\mathcal{A},\mu\in T^1$*, some* α,β,γ,a,b*, and* $|s|\geq|1-2b|\geq 1$. *Then the map* $f:\mathcal{A}\to\mathcal{A}$ *is a* (α,β,γ)*-derivation.*

Corollary 2.11. *If* $k,\theta\in\mathbb{R}$, $0<k<2$, $\theta\geq 0$*, assume the map* $f:\mathcal{A}\to\mathcal{A}$ *satisfies*

$$\begin{aligned}&\|f(a\mu x+by)+f(a\mu x-by)-\mu f(2ax)-bf(y)-bf(-y)\|\\&\geq \|s(f(a\mu x-by)+f(a\mu x+by)-\mu f(2ax))\|,\end{aligned}$$

$$\|\alpha f[x,y]-\beta[f(x),y]-\gamma[x,f(y)]\|\leq \theta(\|x\|^k+\|y\|^k)$$

for $\forall x,y\in\mathcal{A},\mu\in T^1$, $|s|\geq|1-2b|\geq 1$. *Then the map* $f:\mathcal{A}\to\mathcal{A}$ *is a* (α,β,γ)*-derivation.*

3. Conclusions

In this work, the general Drygas functional equation is introduced, the Hyers-Ulam stability of (α, β, γ)-derivations on Lie C^*-algebras is discussed by general Drygas functional inequality with the participation of coefficient a and b.

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Conflict of interest

The authors of this paper declare that they have no conflict of interest.

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