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Collective correlation and screening effects on radiative recombination in classical nonideal plasmas

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Collective correlation and plasma screening effects on the *K*-shell radiative recombination of the free electron with the ion in classical nonideal plasmas are investigated. The radiative recombination cross section is obtained by the principle of detailed balance with the photoionization cross section of the hydrogenic ion in nonideal plasmas. It is found that the collective and plasma screening effects significantly reduce the radiative recombination cross section in nonideal plasmas. It is also found that the collective and screening effects on the recombination cross section is almost independent of the projectile energy.
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Radiative recombination process[1–9] is a subject of special attention in many areas of physics, such as astrophysics, atomic physics, and plasma physics, because it is the inverse process to the photoionization of atoms or ions. This radiative recombination process also has important consequences for x-ray astronomy, since the free electron capture by atoms or ions is the one of the important continuum x-ray emission mechanisms. It has been known that the Debye–Hückel interaction potential describes the properties of low-density plasmas and corresponds to a pair correlation approximation. The plasmas described by the Debye–Hückel model are known as the ideal plasmas, since the average interaction energy between charged particles in the plasmas is small compared to the average kinetic energy of a particle.[10,11] However, multiparticle correlation effects caused by the simultaneous interaction of many charged particles should be taken into account with increasing plasma density. Then, it is necessary to take into account not only short-range collective effects but also long-range Coulomb effects in the case of plasmas with moderate densities and temperatures. In this case, the interaction potential cannot be described by the Debye–Hückel model due to the strong collective effects of nonideal particle interaction.[12,13] Then, the radiative recombination cross sections in classical nonideal plasmas would be different from those in ideal plasmas. Thus, in this paper we investigate the collective and screening effects on the *K*-shell radiative recombination processes by ions in classical nonideal plasmas, since theoretical atomic spectroscopy[14] is essential in the study of plasma parameters.

The inverse process to the radiative recombination is the photoionization process, consisting of photon emission and the absorption of a bound electron into continuum. Since we deal here with binary collisions with the ion X^{+z+1} and X^{+z},

$$X^{+z+1}+e^{-}\leftrightarrow X^{+z}+\hbar\omega, \tag{1}$$

the cross sections (σ_{rr}, radiative recombination; σ_{pi}, photoionization) of both processes have the same dimensions and are mutually related by the principle of detailed balance,

$$\frac{\sigma_{rr}}{\sigma_{pi}}=\frac{j_{\hbar\omega}}{j_{e^-}}\frac{g_{pi}}{g_{rr}}, \tag{2}$$

where $j_{\hbar\omega}(=c/L^3)$ and $j_{e^-}(=v/L^3)$ are the fluxes of photons and electrons for the corresponding channels of the processes, normalized to one particle per given cubic volume (L^3) where c is the speed of light and v is the speed of electron, and ω is the frequency of the emitted photon. In Eq. (2), $g_{pi}[=g_{X^{+z+1}}g_{e^-}4\pi q^2dq/(2\pi)^3]$ and g_{rr} $[=2g_{X^{+z}}4\pi k^2dk/(2\pi)^3]$ are, respectively, the statistical weights of the final states for photoionization and radiative recombination processes, where $g_{X^{+z+1}}$, g_{e^-}, $g_{X^{+z}}$ are statistical weights of the internal state of the particles and q and k are the wave numbers of the electron and the photon, respectively. Here, $g_{X^{+z+1}}=2J'+1$, $g_{e^-}=1$, and $g_{X^{+z}}=2J+1$ where J' and J are the angular momenta of states X^{+z+1} and X^{+z}, and we can take $g_{e^-}=1$, since there is no electron spin in the problem. Using Eq. (2) with the relations $q=mv$ and $k=\omega/c$, we find that the cross section for the radiative recombination of the free electron to the ground state of the ion with charge Z is found to be

$$\sigma_{rr}=\frac{Z^2\alpha^2}{2}\frac{(\bar{E}+\bar{E}_b)^2}{\bar{E}}\sigma_{pi}, \tag{3}$$

where $\alpha(=e^2/\hbar c\approx 1/137)$ is the fine structure constant, $\bar{E}(\equiv mv^2/2Z^2\,\mathrm{Ry})$ is the scaled electron energy, $\bar{E}_b(\equiv E_b/Z^2\,\mathrm{Ry})$ is the scaled binding energy of the electron, $\mathrm{Ry}(=me^4/2\hbar^2\approx 13.6\,\mathrm{eV})$ is the Rydberg constant, and m is the electron rest mass. The radiative recombination into the ground state has the greatest probability, due to the n^{-3} dependence of the radiation spectrum on the principal quantum number n.

In recent papers,[12,15] an integro-differential equation for the effective potential of the particle interaction taking into account the simultaneous correlations of N particles was obtained on the basis of a sequential solution of the chain of Bogolyubov equations for the equilibrium distribution function of particles of classical nonideal plasmas, and an analytic expression for the pseudopotential of the particle inter-

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action in nonideal plasmas was also obtained by the application of the spline approximation. Using the pseudopotential, taking into account the collective and plasma screening effects, the interaction potential $V(r)$ between the free electron and the ion with charge Z in classical nonideal plasmas can be represented as

$$V(r)=-\frac{Ze^2}{r}e^{-r/\Lambda}\frac{1+\gamma f(r)/2}{1+c(\gamma)}, \tag{4}$$

where r is the distance between the projectile electron and the ion, Λ is the Debye length, $f(r)=(e^{-\sqrt{\gamma}r/\Lambda}-1)(1-e^{-2r/\Lambda})/5$, and $\gamma(\equiv e^2/\Lambda kT_e)$ is the nonideality plasma parameter, $c(\gamma)\cong 0.456\gamma-0.108\gamma^2$ is the correlation coefficient, and T_e is the electron temperature. In ideal plasmas, i.e., $\gamma\rightarrow 0$, the pseudopotential [Eq. (4)] goes into the Debye–Hückel interaction potential $V_{\mathrm{DH}}(r)\rightarrow(-Ze^2/r)e^{-r/\Lambda}$. For the recombination of the free electron to the 1s ground state of the ion with charge Z in classical nonideal plasmas with the pseudopotential model [Eq. (4)], the binding energy $E_b(=|E_{1s}|)$ can be obtained by a recent investigation[16] for the screened 1s ground state and 2p excited state energies and screened wave functions using the Ritz variation method

$$E_b(\gamma,\Lambda,Z)=Z'^2\,\mathrm{Ry}=Z^2\,\mathrm{Ry}[1-\delta_{1s}(\gamma,\Lambda,Z)], \tag{5}$$

where Z' is the 1s effective charge, δ_{1s} represents the collective and plasma screening effects on the ground state energy:

$$\delta_{1s}(\gamma,\Lambda,Z)\cong 1-A^2(\gamma)-2A(\gamma)\left(\frac{a_Z}{\Lambda}\right)+\frac{3}{2}\left(\frac{a_Z}{\Lambda}\right)^2\left(1-\frac{2}{5}\gamma^{3/2}\right)-\frac{1}{A(\gamma)}\left(\frac{a_Z}{\Lambda}\right)^3\left(1-\frac{12}{5}\gamma^{3/2}-\frac{3}{5}\gamma^2\right), \tag{6}$$

$A(\gamma)\equiv[1+c(\gamma)]^{-1}$, and $a_Z(\equiv a_0/Z=\hbar^2/Zme^2)$ is the first Bohr radius of the hydrogenic ion with nuclear charge Z. Then the radiative recombination cross section to the 1s ground state is given by

$$\sigma_{1s}^{rr}=(Z^2\alpha^2/2\bar{E})\,(\bar{E}+\Delta_{1s})^2\sigma_{1s}^{pi}, \tag{7}$$

where $\Delta_{1s}[\equiv 1-\delta_{1s}(\gamma,\Lambda,Z)]$ is the screening constant in the 1s ground state and σ_{1s}^{pi} is the 1s photoionization cross section in classical nonideal plasmas described by the pseudopotential model [Eq. (4)]. The K-shell photoionization cross section[17] σ_{1s}^{pi} in nonideal plasmas, including the screening corrections on the bound and continuum states, retardation correction, and Coulomb correction is given by

$$\sigma_{1s}^{pi}(\varepsilon,\gamma,\Lambda,Z)=\frac{2^9\pi^2}{3}\eta_{1s}^{-3}\Delta_{1s}^{1/2}\alpha a_Z^2\varepsilon^{-4}\{1-Z^2\alpha^2\varepsilon^2/(\varepsilon-\Delta_{1s}^{1/2})\}^{-2}\frac{e^{-4(\varepsilon-\Delta_{1s})^{-1/2}\cot^{-1}(\varepsilon-\Delta_{1s})^{-1/2}}}{1-e^{-2\pi(\varepsilon-\Delta_{1s})^{-1/2}}}\left(\frac{\varepsilon-1}{\varepsilon-\Delta_{1s}}\right)\times[\tan^{-1}(\varepsilon-1)^{1/2}]^{-2}|I_{1s}^{(A)}(\varepsilon,\gamma,\Lambda,Z)|^2, \tag{8}$$

where the parameter η_{1s}^{-1} represents the collective and screening effects on the first Bohr radius:

$$\eta_{1s}^{-1}(\gamma,\Lambda,Z)\cong A(\gamma)\left[1-\frac{3}{4}\left(\frac{a_Z}{A(\gamma)\Lambda}\right)^2\left(1-\frac{2}{5}\gamma^{3/2}\right)+\left(\frac{a_Z}{A(\gamma)\Lambda}\right)^3\left(1-\frac{12}{5}\gamma^{3/2}-\frac{3}{5}\gamma^2\right)\right], \tag{9}$$

and $\varepsilon(\equiv\hbar\omega/Z^2\,\mathrm{Ry})$ is the scaled photon energy. Here, $I_{1s}^{(A)}(\varepsilon,\gamma,\Lambda,Z)$ is the dipole acceleration matrix element[17] for the 1s bound state including the collective and plasma screening effects:

$$\begin{aligned}I_{1s}^{(A)}(\varepsilon,\gamma,\Lambda,Z)=&\tan^{-1}\left(\frac{k_f}{\alpha_{1s}^{-1}+\Lambda^{-1}}\right)+\frac{\gamma}{10}\left[\tan^{-1}\left(\frac{k_f}{\alpha_{1s}^{-1}+(\sqrt{\gamma}+1)/\Lambda}\right)-\tan^{-1}\left(\frac{k_f}{\alpha_{1s}^{-1}+\Lambda^{-1}}\right)\right.\\&\left.+\tan^{-1}\left(\frac{k_f}{\alpha_{1s}^{-1}+(\sqrt{\gamma}+1)/\Lambda}\right)+\tan^{-1}\left(\frac{k_f}{\alpha_{1s}^{-1}+3/\Lambda}\right)\right]+\frac{1}{\Lambda}\frac{k_f}{[(\alpha_{1s}^{-1}+\Lambda^{-1})^2+k_f^2]}\\&+\frac{\gamma}{10}\left[\frac{\sqrt{\gamma}+1}{\Lambda}\frac{k_f}{[(\alpha_{1s}^{-1}+(\sqrt{\gamma}+1)/\Lambda)^2+k_f^2]}-\frac{1}{\Lambda}\frac{k_f}{[(\alpha_{1s}^{-1}+\Lambda^{-1})^2+k_f^2]}\right.\\&\left.-\frac{\sqrt{\gamma}+3}{\Lambda}\frac{k_f}{[(\alpha_{1s}^{-1}+(\sqrt{\gamma}+3)/\Lambda)^2+k_f^2]}+\frac{3}{\Lambda}\frac{k_f}{[(\alpha_{1s}^{-1}+3/\Lambda)^2+k_f^2]}\right],\end{aligned} \tag{10}$$

where $\alpha_{1s}[\equiv a_Z\eta_{1s}(\gamma,\Lambda,Z)]$ is the effective Bohr radius and $k_f a_Z[=(\varepsilon-\Delta_{1s})^{1/2}]$ is the scaled wave number of the projectile electron obtained by the energy conservation $\bar{E}+\Delta_{1s}=\varepsilon$. In obtaining Eq. (10), we restricted ourselves to hydrogenic wave functions with $Z\alpha\ll 1$ so that relativistic effect on the bound state wave function was neglected since the relativistic corrections are only of relative order $(Z\alpha)^2$.[18] After some algebra, the $1s$ radiative recombination cross in units of πa_0^2 in nonideal plasmas including the collective and plasma screening effects and the retardation and Coulomb correction effects is then obtained as a function of the scaled electron energy $(\bar{E})$:

$$\sigma_{1s}^{rr}(\bar{E},\gamma,\Lambda,Z)/\pi a_0^2=\frac{2^8\pi}{3}\alpha^3\eta_{1s}^{-3}\Delta_{1s}^{1/2}\frac{(\bar{E}+\Delta_{1s}-1)}{\bar{E}^2(\bar{E}+\Delta_{1s})^2}\left\{1-\frac{Z^2\alpha^2(\bar{E}+\Delta_{1s})^2}{\bar{E}}\right\}^{-2}\frac{e^{-4\bar{E}^{-1/2}\cot^{-1}\bar{E}^{-1/2}}}{1-e^{-2\pi\bar{E}^{-1/2}}}$$
$$\times[\tan^{-1}(\bar{E}+\Delta_{1s}-1)^{1/2}]^{-2}\left\{\tan^{-1}\left(\frac{\bar{E}+\Delta_{1s}-1}{\eta_{1s}^{-1}+a_\Lambda}\right)^{1/2}+\frac{\gamma}{10}\left[\tan^{-1}\left(\frac{\bar{E}+\Delta_{1s}-1}{\eta_{1s}^{-1}+(\sqrt{\gamma}+1)a_\Lambda}\right)^{1/2}\right.\right.$$
$$\left.-\tan^{-1}\left(\frac{\bar{E}^{1/2}}{\eta_{1s}^{-1}+a_\Lambda}\right)-\tan^{-1}\left(\frac{\bar{E}^{1/2}}{\eta_{1s}^{-1}+(\sqrt{\gamma}+3)a_\Lambda}\right)-\tan^{-1}\left(\frac{\bar{E}^{1/2}}{\eta_{1s}^{-1}+3a_\Lambda}\right)\right]$$
$$+\frac{a_\Lambda\bar{E}^{1/2}}{[(\eta_{1s}^{-1}+a_\Lambda)^2+\bar{E}]}+\frac{\gamma}{10}\left[\frac{(\sqrt{\gamma}+1)a_\Lambda\bar{E}^{1/2}}{[(\eta_{1s}^{-1}+(\sqrt{\gamma}+1)a_\Lambda)^2+\bar{E}]}-\frac{a_\Lambda\bar{E}^{1/2}}{[(\eta_{1s}^{-1}+a_\Lambda)^2+\bar{E}]}\right.$$
$$\left.\left.-\frac{(\sqrt{\gamma}+3)a_\Lambda\bar{E}^{1/2}}{[(\eta_{1s}^{-1}+(\sqrt{\gamma}+3)a_\Lambda)^2+\bar{E}]}+\frac{3a_\Lambda\bar{E}^{1/2}}{[(\eta_{1s}^{-1}+3a_\Lambda)^2+\bar{E}]}\right]\right\}^2, \quad (11)$$

where $a_\Lambda(\equiv a_Z/\Lambda)$ is the scaled reciprocal Debye length. This final expression of the radiative recombination cross section [Eq. (11)] in nonideal plasmas is reliable for the range $\Lambda\geqslant 10a_Z$ and $0\leqslant\gamma\leqslant 1$, since the collective and plasma screening effects [Eq. (6)] on the ground state are obtained by the perturbational expansions for the domain $\Lambda\geqslant 10a_Z$ and $0\leqslant\gamma\leqslant 1$ and the collective and plasma screening effects on the continuum Ep state $(2\pi\Delta_{1s}^{1/2}/\varepsilon^{1/2})e^{-4(\varepsilon-\Delta_{1s})^{-1/2}\cot^{-1}(\varepsilon-\Delta_{1s})^{-1/2}}/[1-e^{-2\pi(\varepsilon-\Delta_{1s})^{-1/2}}]$ is also reliable for our interesting domain $\Lambda\geqslant 10a_Z$ and $0\leqslant\gamma\leqslant 1$. Figure 1 shows the radiative recombination cross section for the electron capture into the $1s$ bound state as a function of the scaled electron energy. Figure 2 shows the three-dimensional plot of the radiative recombination cross section as a function of the nonideality plasmas parameter and Debye length when $\bar{E}=100$. As we see in these figures, the collective correlation and plasma screening effects significantly reduce the radiative recombination cross section. It is also found that the collective effect on the recombination cross section is increased as the projectile energy increases. The numerical values of the radiative recombination cross sections in units of πa_0^2 are also listed in Table I. It should be noted that the collective effect reduces the radiative recombination cross section (e.g., ≈13% for $\gamma=0.1$; ≈49% for $\gamma=0.6$). Figure 3 shows the three-dimensional plot of the radiative recombination cross section as a function of the nonideality plasma parameter and projectile energy. As we see in this figure, the radiative recombination cross section decreases with increasing projectile energy. Figure 4 shows the contour plot of the recombination cross section as a function of the nonideality plasma parameter (γ) and the scaled reciprocal Debye length (a_Λ). As we can see in this figure,

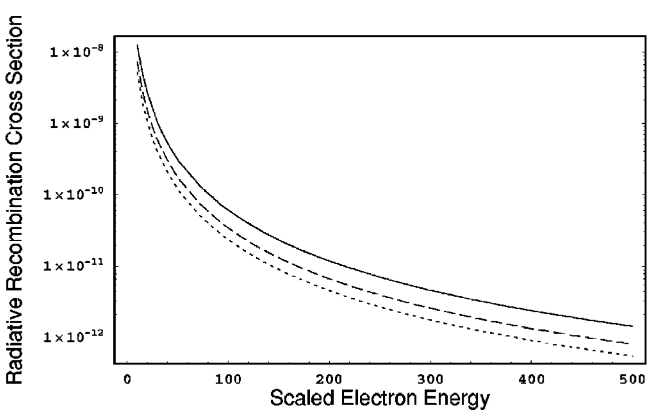

FIG. 1. The $1s$ radiative recombination cross sections σ_{1s}^{rr} in units of πa_0^2 for $Z=2$ and $a_\Lambda=0.1$ as functions of the scaled electron energy: solid line, $\gamma=0$; dashed line, $\gamma=0.5$; dotted line, $\gamma=1$.

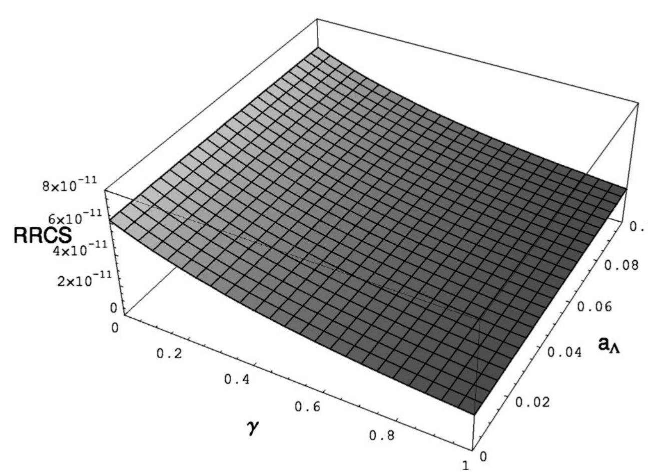

FIG. 2. The three-dimensional plot of the $1s$ radiative recombination cross section (RRCS) σ_{1s}^{rr} in units of πa_0^2 for $Z=2$ as a function of the nonideality plasma parameter (γ) and scaled reciprocal Debye length (a_Λ) when $\bar{E}=100$.

TABLE I. The numerical values of the $1s$ radiative recombination cross sections σ_{1s}^{rr} in units of πa_0^2 for $Z=2$ and $a_\Lambda=0.1$.

γ	$\sigma_{1s}^{rr}(\bar{E}=200)^{a}$ (πa_0^2)	$\sigma_{1s}^{rr}(\bar{E}=400)^{b}$ (πa_0^2)
0	1.182×10^{-11}	2.377×10^{-12}
0.1	1.033×10^{-11}	2.076×10^{-12}
0.2	9.118×10^{-12}	1.832×10^{-12}
0.4	7.310×10^{-12}	1.468×10^{-12}
0.6	6.061×10^{-12}	1.217×10^{-12}
0.8	5.174×10^{-12}	1.038×10^{-12}
1.0	4.533×10^{-12}	9.097×10^{-13}

[a]The radiative recombination cross sections for $\bar{E}=200$.

[b]The radiative recombination cross sections for $\bar{E}=400$.

the nonideal effect plays a more significant role on the recombination cross section than the plasma screening effect. Recently, ionization and recombination rates in dense and nondegenerate carbon plasma[19] were investigated using the generalized kinetic equation including many-body effects and lowering of ionization energies. However, the collective effects on the recombination cross section have not been investigated.

Summarizing, we have obtained the collective and plasma screening effects on the *K*-shell radiative recombination of the free electron to the $1s$ bound state of the hydrogenic ion in classical nonideal plasmas. The charged particle interaction potential in nonideal plasmas is given by the pseudopotential model. The collective and plasma screening effects on the radiative recombination cross section is obtained as a function of the nonideality plasma parameter, Debye length, and projectile energy. It is found that the collective and plasma screening effects significantly reduce the radiative recombination cross section and also that the collective and screening effects are almost independent of the projectile energy. These results provide useful information on radiative recombination processes in classical nonideal plasmas.

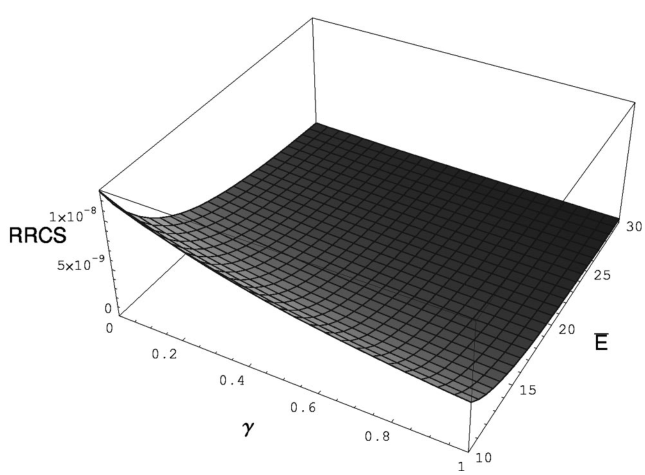

FIG. 3. The three-dimensional plot of the $1s$ radiative recombination cross section (RRCS) σ_{1s}^{rr} in units of πa_0^2 for $Z=2$ as a function of the nonideality plasma parameter (γ) and projectile energy $(\bar{E})$.

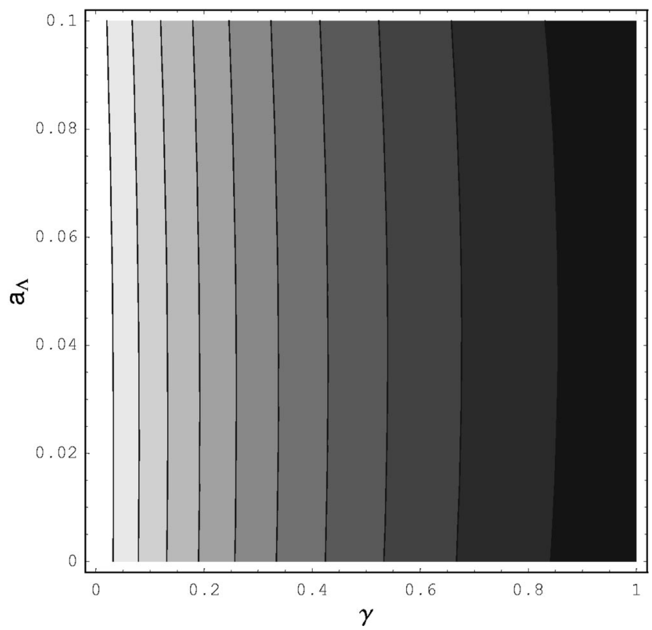

FIG. 4. The contour plot of the $1s$ recombination cross section σ_{1s}^{rr} as a function of the nonideality plasma parameter (γ) and scaled reciprocal Debye length (a_Λ) for $Z=2$ and $\bar{E}=100$.

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