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Maintenance Optimization for Repairable Deteriorating Systems under Imperfect Preventive Maintenance

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Abstract: This study deals with the preventive maintenance optimization problem based on a reliability threshold. The conditional reliability threshold is used instead of the system reliability threshold. Then, the difference between the two thresholds is discussed. The hybrid failure rate model is employed to represent the effect of imperfect preventive maintenance activities. Two maintenance strategies are proposed under two types of reliability constraints. These constraints are set to consider the cost-effective maintenance strategy and to evaluate the balancing point between the expected total maintenance cost rate and the system reliability. The objective of the proposed maintenance strategies is to determine the optimal conditional reliability threshold together with the optimal number of preventive maintenance activities that minimize the expected total maintenance cost per unit time. The optimality conditions of the proposed maintenance strategies are also investigated and shown via four propositions. A numerical example is provided to illustrate the proposed preventive maintenance strategies. Some sensitivity analyses are also conducted to investigate how the parameters of the proposed model affect the optimality of preventive maintenance strategies.

Keywords: preventive maintenance; imperfect repair; optimization; reliability threshold; repairable system

1. Introduction

Most engineering systems experience degradation influenced by the usage environment and operation time length such as aircrafts, high-speed railroads, nuclear power plants, and weapons systems, which leads to frequent failures [1]. Once a failure has occurred in the system, it decreases the availability and reliability of the system, which increase unexpected losses. For example, the availability degradation can increase downtime costs and decrease product quality. Additionally, the reliability degradation can affect the safety of systems and reduce the operating life. Therefore, in many industries, maintenance activities are performed to prevent decreasing availability and reliability of the systems due to severe degradation.

Maintenance activities play an important role in reliability theory because they decrease uncertainty of the failure, reduce the operation cost of the system, and increase the functioning life of the system [2]. In practice, due to increasing complexity of the systems, the cost related to the maintenance activities has increased constantly over the past decades, which attributes 15–70% of production costs [3–5]. Therefore, it is important to establish cost-effective maintenance planning for ensuring higher efficiency, quality production, availability of the system, and reliability of the system.

There are two general types of maintenance activities: corrective maintenance (CM) and preventive maintenance (PM). CM is unscheduled and performed when a system fails, and the system is restored from a failed state to a working state [6]. Most CM employs a minimal repair that returns the failed system to its previous working state after repairing the system. Therefore, the minimal repair has been widely used in numerous maintenance models to address maintenance problems such as the failure of a system. There are many studies covering the minimal repair model [7]. On the other hand, PM activities delay the degradation of a system and reduce operational stress. In traditional PM models, a repaired system can end up in the state of either "as good as new" (i.e., perfect PM) or "as bad as old" (i.e., minimal repair) after the PM activities [8]. Thereafter, a more generalized and realistic approach has been proposed, called the imperfect PM model. The imperfect PM model considers that the system state lies somewhere between "as good as new" and "as bad as old" after PM activities.

Many models describing the effect of imperfect PM activities have been discussed in many existing studies. Malik et al. [9] introduced an imperfect PM model based on the age reduction method. Lin et al. [10] proposed a hybrid hazard rate function that combines the age reduction factor and hazard increasing factor. This model can better reflect the general and realistic situations because the model restores the system effective age to a better state and degrades the hazard rate of the system, after PM activities. El-Ferik and Ben-Daya [11] developed an age-based imperfect PM model that uses the hybrid hazard rate model and considers two types of failure modes. Khatab and Abdelhakim [12] developed an imperfect PM model that is subject to random deterioration based on the hybrid hazard rate using a reliability threshold.

In addition, the PM activity is subcategorized into two types of activities: periodic and non-periodic. Periodic PM activities are repeatedly performed over a fixed period of time T. The periodic PM activity is still widely applied to many PM models because it is mathematically convenient [13]. Shue et al. [14] developed an extended periodic imperfect PM model by considering an age-dependent failure type. Some previous studies [15–17] proposed maintenance policies for a leased system using the periodic imperfect PM model. Moreover, other extended studies on periodic PM have been applied to various fields such as warranty policies, production systems, and used systems [18–21]. However, a periodic PM has a shortcoming in that it is hard to prevent failures that frequently occur in the late period of the life of a system based on the periodic PM. Non-periodic PM models have been proposed to handle this shortcoming, which can prevent the inefficient PM activities [22].

In non-periodic PM models, controlling system reliability is an important issue especially when dealing with repairable deteriorating systems. In practice, maintenance based on reliability threshold provides good guidance for decision-makers who want to attain high performance of maintenance for their operating systems [23]. In addition, reliability of an engineering system is a decision variable that can increase investment in production technology [24]. Zhao [25] proposed a PM policy based on the critical reliability level subject to degradation. Zhou et al. [26] proposed a reliability-centered predictive maintenance policy that is composed of scheduled PM and non-scheduled PM using the reliability threshold. They considered the situation in which the system was continuously monitored. Liao et al. [27] developed a sequential imperfect PM model with a reliability threshold for a continuously monitored system that considered both the operational cost and the breakdown cost. Schutz et al. [16] developed a maintenance strategy for leased equipment based on the reliability threshold. Doostparast et al. [28] developed an optimal PM scheduling considering a certain level of reliability for coherent systems. Lin et al. [29] developed an imperfect PM model with a reliability threshold that considered three reliability constraints to help evaluate the maintenance cost. This model used the system reliability threshold that maintains a survival probability until the system replacement. Khatab et al. [30] developed an imperfect PM model for failure-prone second-hand systems considering the conditional reliability threshold and upgrade action. Khatab [31] improved Liao's model [27] by adding mathematical properties and remedying inconsistencies in the maintenance optimization model.

Traditional PM models using the reliability threshold assume that PM activities are performed when the reliability reaches a threshold value, and that the system is constantly monitored and maintained [29]. However, the system reliability may not be maintained at a constant level when following the foregoing assumption. This is because there are two types of reliability threshold in the PM model: the conditional reliability threshold and the system reliability threshold. The conditional reliability threshold refers to the survival probability of each PM cycle, whereas the system reliability threshold refers to the survival probability until the replacement. The performance of the PM model (e.g., the expected maintenance cost rate and the system operating time until replacement) is not robust to the selection of reliability threshold in constructing the PM model. Therefore, it is important to clearly differentiate between the conditional reliability threshold and the system reliability threshold. This is the motivation of this study.

In addition, it is important to consider the trade-off between reliability and cost when establishing PM models. Some existing models employ reliability thresholds to address the trade-off. However, the existing models assume that the threshold value is same for every PM cycle. This assumption is biased and unrealistic toward reliability. Therefore, the existing models find it difficult to reflect the cost-effectiveness accurately. This is another motivation of this study.

The main contributions of this study are (i) to identify and define the differences between the system reliability threshold and the conditional reliability threshold and (ii) to develop a novel PM model that minimizes the expected maintenance cost rate with consideration of two reliability constraints. Table 1 highlights the contributions of different authors in the field of PM model focusing on the reliability threshold. It can be observed in Table 1 that the literature uses the system reliability threshold more, rather than the conditional reliability threshold. In addition, the literature assumes a fixed level of the reliability threshold.

Table 1. Author contribution in the reliability-based PM model based on reliability constraints, maintenance costs, and imperfect maintenance model.

Research	**Types of Reliability Threshold**	**Reliability Constraints**		**Maintenance Cost**		**Other Cost**	**Imperfect Maintenance Model**		
		Fixed	**Unfixed**	**CM**	**PM**	**Breakdown**	**Age Reduction Factor**	**Hazard Increasing Factor**	**Other**
Zhao [25]	Conditional	√		√	√				√
Zhou et al. [26]	System	√		√	√		√	√	
Liao et al. [27]	Conditional	√		√	√	√	√	√	
Schutz et al. [16]	System	√		√	√		√		
Doostparast et al. [28]	System	√		√	√	√			√
Lin et al. [29]	System	√	√	√	√		√		
Khatab et al. [30]	Conditional	√		√	√	√	√	√	
Khatab [31]	Conditional	√		√	√	√	√	√	
Proposed model	Conditional	√	√	√	√	√	√	√	

In this study, we use the conditional reliability as a criterion of PM activities instead of using system reliability because the conditional reliability threshold can consider the situation in which the system reliability decreases as the frequency of PM activities increase. The hybrid failure rate model is used to represent the effect of imperfect PM activities. In addition, we analyzed the proposed model through two strategies that can help evaluate the trade-off between maintenance cost and system reliability. We discussed the optimal conditions for each strategy via four propositions and one lemma that show the existence and uniqueness of the optimal solution.

The remainder of this paper is organized as follows. Section 2 explains the assumptions to establish the proposed PM model, and derives the function of the expected maintenance cost rate. Section 3 defines two strategies and provides an algorithm to find the optimal solution. Section 4 provides a numerical example to illustrate the proposed PM model and conducts sensitivity analyses to investigate critical elements. Finally, Section 5 discusses the conclusions of this research.

2. Imperfect Preventive Maintenance Model

We consider the situation in which a new system is subject to stochastic deterioration over time and will be replaced at the end of the *N*th PM cycle by a new one. The system experiences PM whenever the conditional reliability threshold reaches a certain level R_i. PM is imperfect and modeled via the hybrid hazard rate model of Lin et al. [10]. Minimal repair is taken for any failures prior to the PM actions. The goal of this study is to determine the optimal number of PM activities and the optimal conditional reliability thresholds for each PM cycle that minimizes the expected maintenance cost rate. To consider the above-mentioned goal and assumptions, we compose the long-term expected maintenance cost rate in this study, which is given as:

$$ECR(\underline{R} = (R_1, R_2, \ldots, R_N), N) = \frac{E[MC] + E[BC]}{E[T]}, \tag{1}$$

where $E[MC]$ denotes the expected total maintenance cost, $E[BC]$ denotes the expected total breakdown cost, and $E[T]$ denotes the expected system replacement cycle. It should be noted that $E[MC]$ is composed of the cost for minimal repair, PM, and replacement. These costs are assumed to be constant and can be obtained from a field investigation. As they may also be considered the expected values for the random variables, it is reasonable to assume $max\ (C_{MR}, C_{PM}, C_{BD}) < C_{RP}$.

2.1. Notations and Assumptions

The notations and detailed assumptions in this study are:

Notations

α	Scale parameter of hazard intensity functions where $\alpha > 0$
β	Deterioration parameter of hazard intensity functions where $\beta > 2$
a_i	Age reduction factor where $0 = a_0 < a_1 < \cdots < a_i < \cdots < 1$
b_i	Hazard increasing factor where $1 = b_0 \le b_1 \le \cdots \le b_i \le \cdots$
x_i	The *i*th PM interval, where $x_0 = 0$ and $x_1 = Y_1$.
Y_i	Effective age of the system before the *i*th PM activity
T_O	System lifetime until replacement
R_i	Conditional reliability threshold where $I = 1, 2, \ldots, N$
C_{MR}	Cost of minimal repair
C_{PM}	Cost of PM activity
C_{RP}	Cost of system replacement
C_{BD}	Cost of system breakdown
$R_{SYS,i}$	System reliability up to the *i*th PM cycle
$R_{CON,i}$	Conditional reliability at the *i*th PM cycle
$R_{RES,i}$	Restored reliability after the *i*th PM activity
$h_i\ (t)$	Hazard intensity functions during the *i*th PM cycle

Assumptions

1. The planning horizon is infinite.
2. The system is replaced at the *N*th PM cycle, and $(N - 1)$ imperfect PMs are performed.
3. The durations of minimal repair, PM, and replacement are negligible.
4. The deterioration process of the system follows the non-homogeneous Poisson process (NHPP), and their hazard intensity function is given as

$$h(t) = \alpha\beta t^{\beta-1}, \text{ where } t \ \geq \ 0. \tag{2}$$

5. The hazard rate of the system varies with the hybrid model after imperfect PM activities that are performed whenever the conditional reliability threshold reaches a certain value.
6. Costs for minimal repair, PM, replacement, and breakdown are assumed constant.

2.2. Hybrid Hazard Rate Model under Imperfect PM

Now, we describe the effect of imperfect PM using the hybrid hazard rate model. This model adjusts the hazard rate of the system using the age reduction factor and the hazard increasing factor after PM activity. Let us consider the situation in which a system's age is reduced by the age reduction factor after PM. If the first PM activity is performed at the effective age Y_1, the age of the system is updated from Y_1 to a_1Y_1. The effective age Y_2 is obtained as the sum of a_1Y_1 and x_2, where x_2 denotes the length of the second PM cycle, and the effective age Y_i can then be generalized as

$$Y_i = x_i + a_{i-1}Y_{i-1}, \tag{3}$$

where $Y_0 = 0$, $x_1 = Y_1$, and a_i denotes the age reduction factor for $0 = a_0 < a_1 < \cdots < a_i < \cdots < 1$.

The hybrid hazard rate model adjusts both the effective age of the system and the hazard rate of the system after PM activity. Now, we describe the hazard rate of the system in each PM cycle. The hazard rate of the system in the first PM cycle is the same as Equation (2) within $[0, Y_1)$. After the first PM, the hazard rate of the system is adjusted from $h(Y_1)$ to $b_1h(a_1Y_1)$, and thus, the hazard rate in the second PM cycle becomes $b_1h(a_1Y_1+t)$ and $t \in [0, x_2)$. After the second PM, the hazard rate of the system is adjusted from $b_1h(Y_2)$ to $b_2b_1h(a_2Y_2)$. The hazard rate in the third PM cycle, then becomes $b_2b_1h(a_2Y_2+t)$ and $t \in [0, x_3)$. According to these procedures, the hazard rate in the ith PM cycle can be generalized as

$$h_i(t) = B_{i-1}h(a_{i-1}Y_{i-1} + t), \tag{4}$$

where $i \geq 1$, $t \in [0, x_i)$, and $B_{i-1} = \Pi_{j=1}^{i-1} b_j$ with $1 = b_0 \leq b_1 \leq \cdots \leq b_i \leq \cdots$.

2.3. Conditional Reliability Threshold

Here, we try to derive the system reliability under imperfect PM. According to the theory of reliability engineering, the reliability is defined as the probability that a system will survive over time period t [32]. The system reliability without PM activity can be defined as:

$$\begin{aligned} R_{SYS}(t) &= \Pr\{T \geq t\} \\ &= \exp\left(-\int_0^t h(u)du\right). \end{aligned} \tag{5}$$

Let $R_{SYS,i}(t)$ be the system reliability at the ith PM cycle, $R_{RES,i-1}(t)$ be the restored reliability after the $(i-1)$th PM, and $R_{CON,i}(t|a_{i-1}Y_{i-1})$ be the conditional reliability at the ith PM cycle. According to the hybrid hazard rate model, after the first PM activity, the effective age is adjusted from Y_1 to a_1Y_1 and the hazard rate of the system is adjusted from $h(Y_1)$ to $B_1h(a_1Y_1)$. Based on these procedures, the restored reliability after the first PM can be inferred as

$$R_{RES,1}(a_1Y_1) = \exp(-B_1H(a_1Y_1)). \tag{6}$$

The conditional reliability at the second PM cycle, $R_{CON,2}(t|a_1Y_1)$, is the survival probability of the system at time t given that the system undergoes the first PM at Y_1; it follows that

$$R_{CON,2}(t|a_1Y_1) = \exp(-B_1\int_{a_1Y_1}^{t} h(u)du), \tag{7}$$

where $t \in [a_1Y_1, Y_2)$. The system reliability after the first PM at time period t is the survival probability over the period $Y_1 + (t - a_1Y_1)$; it can then be computed as the product of the restored reliability and the conditional reliability, which is given as

$$R_{SYS,2}(t) = R_{RES,1}(a_1Y_1)R_{CON,2}(t|a_1Y_1). \tag{8}$$

Based on these procedures, the system reliability after the $(i-1)$th PM cycle can obviously be determined as:

$$R_{SYS,i}(t) = R_{RES,i-1}(a_{i-1}Y_{i-1})R_{CON,i}(t|a_{i-1}Y_{i-1}). \tag{9}$$

According to the aforementioned discussion, we can describe the difference between the conditional reliability threshold and the system reliability threshold through two definitions.

Definition 1. *If imperfect PM activities are performed under the following constraints*

$$R_i = R_{CON,i}(Y_i|a_{i-1}Y_{i-1}), \tag{10}$$

then R_i is called the conditional reliability threshold of the ith PM cycle.

Definition 2. *If imperfect PM activities are performed under the following constraints*

$$\begin{aligned} R_S &= R_{SYS,j}(Y_j) \\ &= R_{RES,j-1}(a_{j-1}Y_{j-1})R_{CON,j}(Y_j|a_{j-1}Y_{j-1}), \end{aligned} \tag{11}$$

then R_S is called a system reliability threshold.

Details of the PM model using the system reliability threshold have been reported in the literature [29]. As the conditional reliability threshold can reflect the situation in which the system reliability decreases as the frequency of PM activities increases while maintaining the reliability above a certain level, it is used to model the present work. Moreover, as the system undergoes PM whenever the conditional reliability threshold reaches the certain level, the corresponding reliability constraints can be given as:

$$R_i = \exp\left(-B_{i-1}\int_{a_{i-1}Y_{i-1}}^{Y_i} h(t)dt\right), \tag{12}$$

for $i = 1, 2, \ldots, N$. Equation (12) is inferred as:

$$\ln R_i = -B_{i-1}\int_{a_{i-1}Y_{i-1}}^{Y_i} h(t)dt. \tag{13}$$

Solving Equation (13) with respect to Y_i, the effective age at the ith PM activity can be determined as:

$$Y_i = \left(\frac{-B_{i-1}^{-1}\ln R_i - \sum_{j=1}^{i-1}\left(\prod_{k=j}^{i-1}\rho_k^{\beta}\right)B_{j-1}^{-1}\ln R_j}{\alpha}\right)^{1/\beta}, \tag{14}$$

where $i = 2, \ldots, N$ and $Y_1 = (-\ln R_1/\alpha)^{1/\beta}$.

2.4. Long-Term Expected Maintenance Cost Rate

The expected total maintenance cost until the Nth PM cycle is composed of the replacement cost, PM cost, and minimal repair cost, which is given as:

$$\begin{aligned} E[MC] &= C_{RP} + (N-1)C_{PM} + C_{MR}\sum_{i=1}^{N}\left(\int_0^{Y_i} h_i(t)dt\right) \\ &= C_{RP} + (N-1)C_{PM} + C_{MR}\sum_{i=1}^{N}\left(B_{i-1}\int_{a_{i-1}Y_{i-1}}^{Y_i} h(t)dt\right). \end{aligned} \tag{15}$$

Through Equation (13), Equation (15) becomes:

$$E[MC] = C_{RP} + (N-1)C_{PM} - C_{MR}\sum_{i=1}^{N} \ln R_i. \tag{16}$$

Real engineering systems suffer from frequent breakdowns after long-term use due to the severe degradation of the system. At this point, it may be more economical to replace the system rather than performing PM. Liao et al. [27] pointed out the importance of breakdown cost when constructing the PM models and used the breakdown cost in their PM model. They considered that the breakdown costs are charged once per PM cycle. Unlike the computation approach used in Liao et al. [27], Khatab [31] considered the breakdown cost that occurs whenever the system undergoes maintenance activities, such as a minimal repair, PM, and replacement. He discussed that the breakdown costs are computed as the product of the sum of all maintenance activities until the Nth PM cycle and the value of the breakdown cost. This study employs the approach proposed by Khatab [31] in calculating the breakdown cost. The total breakdown cost in this study is given as

$$\begin{aligned} E[BC] &= C_{BD}\left(N + \sum_{i=1}^{N}\int_{0}^{Y_i} h_i(t)dt\right) \\ &= C_{BD}\left(N - \sum_{i=1}^{N} \ln R_i\right). \end{aligned} \tag{17}$$

The replacement cycle of the system can be calculated by the sum of the durations up to the Nth PM cycle, which is given as:

$$T_O = Y_N + \sum_{j=1}^{N-1} (1-a_j)Y_j. \tag{18}$$

Through Equations (15)–(18), the long-term expected maintenance cost rate becomes:

$$ECR(\underline{R} = (R_1, R_2, \ldots, R_N), N) = \frac{C_{RP} + (N-1)C_{PM} - C_{MR}\sum_{i=1}^{N} \ln R_i + C_{BD}\left(N - \sum_{i=1}^{N} \ln R_i\right)}{Y_N + \sum_{j=1}^{N-1} (1-a_j)Y_j}. \tag{19}$$

3. Maintenance Optimization

In this section, we introduce the proposed PM model by discussing two strategies with different reliability constraints. The optimization problem of the proposed PM model is to find the decision variables that minimize the expected maintenance cost rate. In this optimization problem, the decision variables are defined as the optimal number of PM activities and the optimal conditional reliability thresholds.

Strategy 1: The PM activities are performed with the same level of conditional reliability threshold, which is given as:

$$R_1 = \exp\left(-B_{i-1}\int_{a_{i-1}Y_{i-1}}^{Y_i} h(t)dt\right), \text{for } i = 1, 2, \ldots, N. \tag{20}$$

Strategy 2: The PM activities are performed at the different values of conditional reliability thresholds on all PM cycles. This PM strategy relaxes the reliability constraints of strategy 1 and is assumed to follow Equation (12). The system reliability of strategy 2 is lower than that of strategy 1, but it can lower the expected total maintenance cost rate.

3.1. Strategy 1: Same Level of Conditional Reliability Threshold

This strategy assumes that the conditional reliability thresholds on all PM cycles have the same value. The PM activity of this strategy is performed whenever the conditional reliability threshold reaches a certain level. The decision variables in this strategy are the optimal number of PM activities and the optimal conditional reliability threshold. These values minimize the expected total maintenance cost rate. In this strategy, the reliability constraints for performing PM activities follow Equation (20). Solving Equation (20) with respect to Y_i, the effective age at the ith PM activity can be determined as:

$$Y_i = \left(\frac{-B_{i-1}^{-1} \ln R_1 - \sum_{j=1}^{i-1} \left(\prod_{k=j}^{i-1} \rho_k^{\beta} \right) B_{j-1}^{-1} \ln R_1}{\alpha} \right)^{1/\beta}, \quad (21)$$

where $i = 2, \ldots, N$ and $Y_1 = (-\ln R_1/\alpha)^{1/\beta}$.

The expected total maintenance cost rate of strategy 1 becomes:

$$ECR_1(R_1, N) = \frac{C_{RP} + (N-1)C_{PM} - NC_{MR} \ln R_1 + C_{BD}(N - N \ln R_1)}{Y_N + \sum\limits_{j=1}^{N-1} (1 - a_j) Y_j}. \quad (22)$$

Now, we try to find the optimal number of PM activities that minimizes Equation (22). The inequalities are:

$$ECR_1(R_1, N-1) > ECR_1(R_1, N) \text{ and } ECR_1(R_1, N) \leq ECR_1(R_1, N+1), \quad (23)$$

if and only if

$$K_1(N-1) < C_{RP} - C_{PM} \text{ and } K_1(N) \geq C_{RP} - C_{PM}, \quad (24)$$

where

$$K_1(N) = \left(\frac{\sum_{i=1}^{N} x_i}{x_{N+1}} - N \right) (C_{PM} + C_{BD} - (C_{MR} + C_{BD}) \ln R_1). \quad (25)$$

Inequalities (23) and (24) are a necessary condition to find where Equation (22) is a convex function with respect to N when R_1 is fixed. Through the following Proposition 1, we can see that there exists an optimal finite and unique N^*.

Proposition 1. *When R_1 is fixed, if Inequalities (22) and (23) are satisfied, Equation (22) is a convex function and there exists an optimal finite and unique N^* that minimizes Equation (22).*

Proof. See Appendix A. □

Through following Proposition 2, we discuss the optimal value of the conditional reliability threshold when N is fixed.

Proposition 2. *When N is fixed, the optimal value for the conditional reliability threshold is given as:*

$$-\ln R_1^* = \frac{N(C_{PM} + C_{BD}) - C_{PM} + C_{RP}}{(\beta - 1)N(C_{MR} + C_{BD})}. \quad (26)$$

Proof. See Appendix A. □

The optimal solutions for strategy 1 can be obtained via the following algorithm that is given as the pseudo-code and is followed as Table 2.

Table 2. Algorithm to obtain the optimal solution for strategy 1.

Algorithm Pseudo-Code for the Minimization of Equation (22)
Input data: C_M, C_R, C_P, C_{BD}, α, β, a_i, b_i;
Set $N = 1$, $M = 2$;
Compute $R_1(N)$, $R_1(M)$ given in Equation (26);
Compute $ECR_1(N\|R_1(N))$, $ECR_1(M\|R_1(M))$ given in Equation (22)
while ($ECR_1(R_1(N), N) > ECR_1(R_1(M), M)$)
$N = N + 1$, $M = M + 1$;
Compute $R_1(N)$, $R_1(M)$ given in Equation (26);
Compute $ECR_1(R_1(N), N)$, $ECR_1(R_1(M), M)$ given in Equation (22);
end while
Set $N^* = N$, $R^*_1 = R_1$ (N^*);
Using N^* and R^*_1, compute $Y_i$given in Equation (21);

3.2. Strategy 2: Different Level of Conditional Reliability Threshold

Unlike strategy 1, this strategy assumes that the conditional reliability thresholds on all PM cycles have difference values. The structural properties of this strategy follow Section 2. The aim of this strategy is to find the optimal number of PM activities N^* and the optimal conditional reliability thresholds $\underline{R} = (R_1, R_2, \ldots, R_N)$. The expected total maintenance cost rate is the same as Equation (19). Solving Equation (14) with respect to Y_i, the effective age at the ith PM activity can be determined as:

$$Y_i = \left(\frac{-B_{i-1}^{-1} \ln R_i - \sum_{j=1}^{i-1} \left(\prod_{k=j}^{i-1} \rho_k^{\beta} \right) B_{j-1}^{-1} \ln R_j}{\alpha} \right)^{1/\beta}, \tag{27}$$

where $i = 2, \ldots, N$ and $Y_1 = (-\ln R_1/\alpha)^{1/\beta}$.

The result of following Lemma 1 will be used in Propositions 3 and 4. The lemma derives the N conditional reliability thresholds to a single conditional reliability threshold.

Lemma 1. *When N is fixed, the conditional reliability thresholds that minimize Equation (17) have the following relationship:*

$$-\ln R_i = \begin{cases} B_{i-1}\left(\frac{A_{i-1}}{A_i} - a_{i-1}^{\beta}\right)\left(\frac{A_1}{A_{i-1}}\right)(-\ln R_1) & \text{, if } 2 \le i \le N-1, \\ B_{N-1}\left(A_{N-1} - a_{N-1}^{\beta}\right)\left(\frac{A_1}{A_{N-1}}\right)(-\ln R_1) & \text{, if } i = N, \end{cases} \tag{28}$$

where

$$A_i = \left[\frac{1}{B_{N-1}} \left(\frac{B_{i-1} - a_i^{\beta} B_i}{1 - a_i} \right) \right]^{\beta/(\beta-1)} \quad \text{for all } i. \tag{29}$$

Proof. See Appendix A. □

Through the result of the following Lemma, Equation (19) becomes:

$$ECR_2(R_1, N) = \frac{(C_{RP} + C_{BD}) + (N-1)(C_{PM} + C_{BD}) - (C_{MR} + C_{BD}) Mr(N) \ln R_1}{Y_N + \sum\limits_{j=1}^{N-1} (1 - a_j) Y_j}, \tag{30}$$

where

$$Mr(N) = 1 + \sum_{i=2}^{N-1} B_{i-1} \left(\frac{A_{i-1}}{A_i} - a_{i-1}^{\beta} \right) \left(\frac{A_1}{A_{i-1}} \right) + B_{N-1} (A_{N-1} - a_{N-1}^{\beta}) \left(\frac{A_1}{A_{N-1}} \right), \tag{31}$$

where $Mr(1) = 1$ and $Mr(2) = 1 + B_1(A_1 - a_1^{\beta})$.

Through following Proposition 3, we can see that there exists an optimal finite and unique N^*.

Proposition 3. *When R_1 is fixed, N^* that satisfies Inequality (32) is the unique and finite optimal value that minimizes Equation (30).*

$$K_2(N-1) < C_{RP} - C_{PM} \text{ and } K_2(N) \geq C_{RP} - C_{PM}, \tag{32}$$

where

$$K_2(N) = \left(\frac{\sum_{i=1}^{N} x_i}{x_{N+1}} - N\right)(C_P + C_{BD}) - (C_M + C_{BD})\frac{\ln R_1}{x_{N+1}}\left(Mr(N+1)\sum_{i=1}^{N} x_i - Mr(N)\sum_{i=1}^{N+1} x_i\right). \tag{33}$$

Proof. See Appendix A. □

The following Proposition provides the optimal conditional reliability threshold of strategy 2.

Proposition 4. *When N is fixed, the solution to the optimal conditional reliability threshold that minimizes Equation (30) is given as:*

$$-\ln R_1^* = \frac{N(C_{PM} + C_{BD}) - C_{PM} + C_{RP}}{(\beta - 1)Mr(N)(C_{MR} + C_{BD})}. \tag{34}$$

Proof. See Appendix A. □

The optimal solutions for strategy 2 can be obtained via the following algorithm that is given as pseudo-code and is followed as Table 3.

Table 3. Algorithm to obtain the optimal solution for strategy 2.

Algorithm Pseudo-Code for the Minimization of Equation (30)
Input data: C_M, C_R, C_P, C_{BD}, α, β, a_i, b_i;
Set $N = 1$, $M = 2$;
Compute $R_1(N)$, $R_1(M)$ given in Equation (34);
Compute $Mr(N)$, $Mr(M)$ given in Equation (31);
Compute $ECR_2(R_1(N), N)$, $ECR_2(R_1(M), M)$ given in Equation (30);
while ($ECR_2(R_1(N), N) > ECR_2(R_1(M), M)$)
$N = N + 1$, $M = M + 1$;
Compute $R_1(N)$, $R_1(M)$ given in Equation (34);
Compute $Mr(N)$, $Mr(M)$ given in Equation (31);
Compute $ECR_2(R_1(N), N)$, $ECR_2(R_1(M), M)$ given in Equation (30);
end while
Set $N^* = N$, $R^*_1 = R_1(N^*)$;
Using N^* and R^*_1, compute $R_N^* = (R_1, \ldots, R_N^*)$ given in Equation (28);
Using N^* and R^*_1, compute Y_i given in Equation (27);

4. Numerical Illustration

In this section, we illustrate the proposed PM strategy based on a numerical example. To conduct the numerical illustration, we used Python 3.7. The procedures of the numerical example are followed as:

Step 1. Input data related to maintenance costs and parameters of hybrid hazard rate function.

Step 2. Compute the conditional reliability threshold and the expected maintenance cost rate.

Step 3. Repeat the step 2 until satisfying $ECR_k(R_1(N), N) > ECR_k(R_1(M), M)$ for $k = 1,2$.

Step 4. Compute PM schedule, the optimal number of PM activities, and the optimal conditional reliability threshold.

In the numerical illustration, two additional PM strategies are considered that can help evaluate the effect of the trade-off between reliability and cost. In the additional PM strategies, the conditional reliability threshold is provided to the decision maker, in which this is given as R_1 = 0.9 (PM strategy 3) and R_1 = 0.3 (PM strategy 4). Sensitivity analyses are also conducted to find how the parameters affect the proposed PM strategy. Table 4 sets out the parameters for conducting the numerical illustration. The functions of the age reduction factor and the hazard increasing factor are set as follows:

$$a_i = \frac{i}{2i+2},\ b_i = \frac{13i+4}{12i+4}. \tag{35}$$

Table 4. Setting of parameters.

C_{MR}/C_{PM}	C_{RP}/C_{PM}	C_{BD}/C_{PM}	α	β
3.0	5.0	0.3	2.6	3.2

4.1. Results of the Numerical Illustration

Figure 1 shows the curves for the expected total maintenance cost rate under the increasing PM activities. Table 5 summarizes the results of this study.

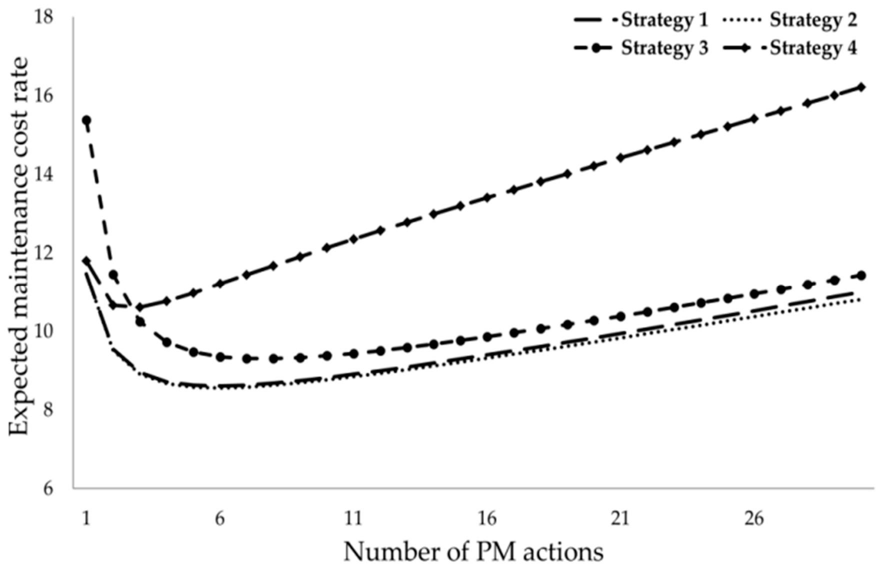

Figure 1. Cost curve under increasing the number of preventive maintenance (PM) activities.

Table 5. The optimal solutions and their system performances.

No. PM	PM Strategy 1		PM Strategy 2		PM Strategy 3		PM Strategy 4	
	x_i	R_S (R^*_1)	x_i	R_S (R^*_1)	x_i	R_S (R^*_1)	x_i	R_S (R^*_1)
1	0.4933	0.7627 (0.7627)	0.5194	0.7265 (0.7265)	0.3672	0.9000 (0.9000)	0.7862	0.3000 (0.3000)
2	0.3626	0.7601 (0.7627)	0.3534	0.7637 (0.7668)	0.2699	0.8988 (0.9000)	0.5779	0.2955 (0.3000)
3	0.3164	0.7561 (0.7627)	0.2968	0.7841 (0.7901)	0.2356	0.8969 (0.9000)	0.5043	0.2886 (0.3000)
4	0.2905	0.7528 (0.7627)	0.2655	0.7981 (0.8072)	0.2163	0.8955 (0.9000)		
5	0.2728	0.7504 (0.7627)	0.2441	0.8091 (0.8196)	0.2031	0.8943 (0.9000)		
6	0.2593	0.7485 (0.7627)	0.3271	0.6660 (0.6753)	0.1931	0.8934 (0.9000)		
7					0.1848	0.8928 (0.9000)		
8					0.1777	0.8922 (0.9000)		
N^*	6		6		8		3	
T_O	1.9950		2.0065		1.8477		1.8684	
R^*_1	0.7627		0.7265					
EMC (N^*, R^*_1)	8.6034		8.5542		9.2988		10.6077	

From the results of this study, we can see that the larger the conditional reliability threshold that we set, the shorter PM cycle and the more PM activities the optimal solution needs. As shown in Table 5, the optimal value N of PM strategy 3 is higher than the optimal ones obtained from strategies 1, 2, or 4. In addition, as R_1 for the PM strategy 3 is set to a high value, the reliability of the system remains high. However, this leads to high PM cost and breakdown cost, since there are frequent PM activities. As shown in Figure 1, the expected total maintenance cost rate of PM strategy 3 is higher than that of PM strategies 1 and 2.

In the cases where the conditional reliability threshold is set to a low value, the length of the PM cycle increases. The result of this is many system breakdowns and only minimal repairs are allowed, and thus, the expected total maintenance cost rate dramatically increases over time. Moreover, we can observe that a trade-off between the reliability threshold and the expected total maintenance cost rate. Therefore, appropriate conditional reliability thresholds are required to establish a cost-effective maintenance strategy.

Unlike the results of PM strategies 3 and 4, PM strategies 1 and 2 focus on finding a balance between the conditional reliability threshold and the expected total maintenance cost rate. Hence, we can see that the results of PM strategies 1 and 2 are more cost-effective than the results of PM strategies 3 and 4. Furthermore, we can see that PM strategy 2 is more cost-effective than PM strategy 1 because PM strategy 2 relaxed the reliability constraints of PM strategy 1. From Table 5, we can see that the optimal conditional reliability thresholds of PM strategy 2 tend to increase slightly except for the value that corresponds to N^*. This means that it is more reasonable to perform PM activities frequently than to replace the system up to ($N^* - 1$) PM cycles. Since the system is then replaced at the N^*th PM activity, the conditional reliability threshold of the last cycle decreases. This implies that operating the system for as long as possible in the last PM cycle is more efficient than performing imperfect PM activity.

4.2. Sensitivity Analyses

Several sensitivity analyses were conducted to investigate how the parameters affected the proposed models. First, to determine how the parameters of the hazard intensity function influence the proposed models, sensitivity analysis was conducted for the situation in which α was adjusted from 1.92 to 2.88 and β was adjusted from 2.88 to 4.32. The results are summarized in Table 6. It is shown that changing of α had no effect on the optimal solutions because the change in α could not reflect the trend in the deterioration process. However, since α was used to calculate the PM cycle, the system lifetime and the expected total maintenance cost rate were affected by the change in α.

The change in β affected the optimal solutions unlike the results of the sensitivity analyses for α because β reflects the shape of the hazard intensity. For example, when β received a low value, the hazard intensity increased from the initial period, resulting in shorter PM intervals. As shown in

Table 6, N^* decreased as the value of β was low. Moreover, $R_1{}^*$ increases as β increases, which implies that the system degradation accelerates when β is high.

Table 6. Results of sensitivity analysis related to the failure rate's parameters.

α	β	Strategy 1					Strategy 2				
		R^*_1	N^*	R_S	T_O	EMC_1 (N^*, R^*_1)	R^*_1	N^*	R_S	T_O	EMC_2 (N^*, R^*_1)
2.08	3.20	0.7627	6	0.7485	2.1391	8.0239	0.7265	6	0.6660	2.1514	7.7980
2.34	3.20	0.7627	6	0.7485	2.0618	8.3248	0.7265	6	0.6660	2.0736	8.2771
2.60	3.20	0.7627	6	0.7485	1.9550	8.6034	0.7265	6	0.6660	2.0065	8.5542
2.86	3.20	0.7627	6	0.7485	1.9364	8.8635	0.7265	6	0.6660	1.9476	8.8128
3.12	3.20	0.7627	6	0.7485	1.8845	9.1079	0.7265	6	0.6660	1.8953	9.0557
2.60	2.56	0.6650	5	0.6351	1.7113	10.0687	0.6251	5	0.5486	1.7213	10.0101
2.60	2.88	0.7128	5	0.6933	1.7342	9.2751	0.6831	5	0.6090	1.7441	9.2224
2.60	3.20	0.7627	6	0.7485	1.9550	8.6034	0.7265	6	0.6660	2.0065	8.5542
2.60	3.52	0.7894	6	0.7798	2.0492	8.0434	0.7595	6	0.7021	2.0604	7.9998
2.60	3.84	0.8190	7	0.8119	2.3400	7.5694	0.7857	7	0.7379	2.3523	7.5298

In addition, we conducted sensitivity analyses to investigate how the change in related costs for maintenance activity affected the proposed model. The results are summarized in Table 7. As the replacement cost increases or the PM cost decreases, the optimal number of PM activities increases. This implies that it is more economical to perform PM activities than to replace the system. As the minimal repair cost increases, the optimal conditional reliability threshold increases. However, the optimal number of PM activities remains unchanged. As the conditional reliability threshold increases, the length of the PM cycle shortens, thereby increasing the expected total maintenance cost rate. Indeed, as shown in Table 7, the optimal conditional reliability threshold and the expected total maintenance cost rate increases as the minimal repair cost increases. Moreover, for both PM strategies, the value of the optimal conditional reliability threshold is significantly sensitive to changes in the minimal repair cost. Therefore, decision-makers should pay close attention to the estimation of minimal repair cost.

Table 7. Results of sensitivity analysis related to maintenance costs.

C_{MR}	C_{PM}	C_{RP}	Strategy 1					Strategy 2				
			R^*_1	N^*	R_S	T_O	EMC_1 (N^*, R^*_1)	R^*_1	N^*	R_S	T_O	EMC_2 (N^*, R^*_1)
2.40	1.00	5.00	0.7181	6	0.7018	2.1241	8.0805	0.6766	6	0.6085	2.1363	8.0342
2.70	1.00	5.00	0.7423	6	0.7271	2.0553	8.3510	0.7036	6	0.6395	2.0671	8.3032
3.00	1.00	5.00	0.7627	6	0.7485	1.9550	8.6034	0.7265	6	0.6660	2.0065	8.5542
3.30	1.00	5.00	0.7801	6	0.7668	1.9415	8.8406	0.7461	6	0.6890	1.9526	8.7900
3.60	1.00	5.00	0.7952	6	0.7826	1.8935	9.0645	0.7631	6	0.7090	1.9044	9.0126
3.00	0.80	5.00	0.7912	7	0.7772	2.1434	8.0756	0.7472	7	0.6991	2.1562	8.0276
3.00	0.90	5.00	0.7715	6	0.7577	1.9682	8.3511	0.7364	6	0.6776	1.9795	8.3033
3.00	1.00	5.00	0.7627	6	0.7485	1.9550	8.6034	0.7265	6	0.6660	2.0065	8.5542
3.00	1.10	5.00	0.7406	5	0.7274	1.7924	8.8454	0.7169	5	0.6453	1.8023	8.7970
3.00	1.20	5.00	0.7325	5	0.7189	1.8127	9.0673	0.7082	5	0.6350	1.8227	9.0177
3.00	1.00	4.00	0.7697	5	0.7577	1.7170	8.0477	0.7482	5	0.6827	1.7265	8.0037
3.00	1.00	4.50	0.7592	5	0.7467	1.7448	8.3365	0.7368	5	0.6691	1.7544	8.2910
3.00	1.00	5.00	0.7627	6	0.7485	1.9550	8.6034	0.7265	6	0.6660	2.0065	8.5542
3.00	1.00	5.50	0.7652	7	0.7497	2.2347	8.8521	0.7168	7	0.6643	2.2481	8.7994
3.00	1.00	6.00	0.7577	7	0.7418	2.2601	9.0746	0.7081	7	0.6543	2.2736	9.0206

5. Conclusions

In this study, we established a cost-effective PM strategy for a repairable system subject to stochastic deteriorations. Two PM strategies were proposed and modeled by two reliability constraints that help evaluate the trade-off between the expected total maintenance cost rate and the system reliability. Strategy 1 constricted the conditional reliability threshold with the same level, whereas

Strategy 2 relaxed the reliability constraints of Strategy 1 by using different level of the conditional reliability threshold. Moreover, this study discussed the difference between the conditional reliability threshold and the system reliability threshold via two definitions. The conditional reliability threshold was used as a condition variable that represents the decreasing system reliability as the number of PM activities increases.

This study employed a hybrid hazard rate model to represent imperfect PM activities. The function of the expected total maintenance cost rate was determined, including the costs of PM activity, replacement, minimal repair, and breakdown. We provided the structural properties of the proposed PM strategies via four propositions and proved their existence and uniqueness of the propositions. Two algorithms were proposed to find optimal solutions. A numerical example was conducted to illustrate the proposed PM strategy. Sensitivity analyses were also conducted to investigate how the parameters of the proposed PM strategy affect the optimal solutions.

5.1. Contributions to Theory

The contributions of this study are as follows:

(1) This study identifies and defines the differences between the system reliability threshold and the conditional reliability threshold. This contributes to the reliability theory.

(2) This study develops a cost-effective PM model using the conditional reliability threshold that considers the decreasing system reliability as the number of PM activities increases, which can relax the trade-off between the reliability and the cost.

(3) This study provides the optimal conditions for each strategy through four propositions and one lemma that show the uniqueness and existence of the optimal solution.

5.2. Implications for the Decision-Makers

Decision makers need to decide whether they undertake all the maintenance activities to keep the system in good condition or reduce maintenance activities in planning PM policy to cut down the costs. For example, if they reduce the costs related to maintenance activities in a PM strategy, it is difficult to obtain high quality of the system due to the insufficient PM activities. On the other hand, frequent PM activities result in higher maintenance costs and poorer system availability. The proposed model can be a good alternative in solving these problems. Based on the proposed model, they can avoid both the extremely frequent maintenance activities and the insufficient maintenance ones effectively.

5.3. Limitations and Further Research

This study has some limitations. First, the parameters of hazards rate function are not estimated based on data but predetermined manually. Second, we need more constraints to apply the model in real word applications because the proposed model is applied to complex and expensive systems. For example, the optimal PM strategies in this work may be expanded to the system under lease, considering the various constraints for the reliability such as the maximum reliability, the minimum reliability, and the reliability at the end of lease. To tackle these limitations, we can estimate the parameters based on the data and add more constraints to handle the complex and difficult situations in real world applications.

For the next step of the research, we can also extend the proposed model to apply to many different areas that cover multi component systems. The expected result is a novel algorithm that not only solves the integrated optimization problem of the proposed PM strategies but also maintains production costs and requirements. In addition, another next step of this study includes solving the two dimensional warranty problem of the proposed model while considering the warranty cost and the numerous uncertainties.

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Appendix A

Appendix A.1 Proof of Proposition 1

When R_1 is fixed, the expected total maintenance cost rate becomes a univariate function. Inequalities (23)–(24) are a necessary condition to obtain the optimal number of PM activities. Inequality (24) is obtained from substituting Equation (22) into Inequality (23). If N^* satisfies Inequality (24), it becomes a finite and unique optimal solution such that $Min(ECR_1(N), N = 1,2, \ldots) = ECR_1(N^*)$. Now, to prove Proposition 1, we recall $K_1(N)$, which is as follows:

$$K_1(N) = \left(\frac{\sum_{i=1}^{N} x_i}{x_{N+1}} - N\right)(C_{PM} + C_{BD} - (C_{MR} + C_{BD}) \ln R_1). \quad \text{(A1)}$$

The left hand side of $K_1(N)$ increases as N increases because x_{N+1} decrease to zero. Moreover, the right hand side of $K_1(N)$ is a positive because $-\ln R_1$ is a positive value. Hence, $K_1(N)$ in Inequality (24) is the increasing function with respect to N, and N^* that satisfies Inequality (24) is then a finite and unique optimal solution that minimizes Equation (22).

Appendix A.2 Proof of Proposition 2

When the number of PM activities is fixed, the condition to obtain the optimal conditional reliability threshold that minimizes Equation (22) is as follows:

$$\frac{dECR_1(R_1|N)}{dR_1} = 0. \quad \text{(A2)}$$

Solving Equation (A2) with R_1, we obtained Equation (26). Moreover, Equation (26) is a finite and unique solution minimizes Equation (22) because $d^2ECR_1(R_1|N)/d^2R_1 > 0$.

Appendix A.3 Proof of Lemma 1

When the number of PM activities is fixed, the conditions to obtain the optimal conditional reliability threshold that minimizes Equation (17) are as follows:

$$\begin{cases} \frac{dECR_2(\underline{R}=(R_1,R_2,\cdots,R_N)|N)}{dR_1} = 0 \\ \vdots \\ \frac{dECR_2(\underline{R}=(R_1,R_2,\cdots,R_N)|N)}{dR_N} = 0 \end{cases}. \quad \text{(A3)}$$

Equation (A3) yields the following relationships:

$$\alpha\beta(C_m + C_{bd})B_{i-1}\left(\frac{dT_o}{dR_i}\left(Y_N + \sum_{i=1}^{N-1}(1 - a_i Y_i)\right)\right) = ECR_2(\underline{R} = (R_1, R_2, \cdots, R_N)|N), \quad \text{(A4)}$$

for $i = 1, 2, \ldots, N$. Solving the simultaneous equation in Equation (A4), we have the following relationships:

$$\begin{aligned} S(N) &= \left[\frac{1}{B_{N-1}} \frac{B_{N-2}-a_{N-1}^{\beta}B_{N-1}}{1-a_{N-1}}\right]^{\frac{\beta}{\beta-1}} S(N-1) \\ &\vdots \\ &= \left[\frac{1}{B_{N-1}} \frac{B_1-a_2^{\beta}B_2}{1-a_2}\right]^{\frac{\beta}{\beta-1}} S(2) \\ &= \left[\frac{1}{B_{N-1}} \frac{1-a_1^{\beta}B_1}{1-a_1}\right]^{\frac{\beta}{\beta-1}} S(1) \end{aligned} \tag{A5}$$

where $S(1) = -\ln R_1$ and

$$S(i) = -B_{i-1}^{-1}\ln R_i - \sum_{j=1}^{i-1}\left(\prod_{k=j}^{i-1}\rho_k^{\beta}\right)B_{j-1}^{-1}\ln R_j, \text{ for } i = 2, 3, \ldots, N. \tag{A6}$$

Using the relationships in Equation (A5), we obtained Equation (28).

Appendix A.4 Proof of Proposition 3

Using the results of Lemma 1, Equation (17) becomes Equation (30). To prove Proposition 3, we redefine $K_2(N)$ in Inequality (33) as follows:

$$K_2(N) = A(N) + B(N)C(N), \tag{A7}$$

where

$$\begin{aligned} A(N) &= \left(\frac{\sum_{i=1}^{N} x_i}{x_{N+1}} - N\right)(C_{PM} + C_{BD}), \\ B(N) &= -(C_{MR} + C_{BD})\frac{\ln R_1}{x_{N+1}}, \\ C(N) &= \sum_{i=1}^{N} x_i(Mr(N+1) - Mr(N)) - Mr(N)x_{N+1}. \end{aligned} \tag{A8}$$

$A(N)$ and $B(N)$ increase as N increase, and $C(N)$ increases as N increases because x_{N+1} decreases to zero. Hence, there exists a finite and unique N^* that minimizes Equation (30).

Appendix A.5 Proof of Proposition 4

Using the results of Lemma 1, Equation (17) becomes Equation (30), and then, Equation (34) is a finite and unique solution that minimizes Equation (30) because $d^2ECR_2(R_1|N)/d^2R_1 > 0$.

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