Walking Pattern Generation using Approximate Unstable Zero Cancelation and Its Compensation Method

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***Abstract*— This paper suggests a walking pattern generation method using an approximate unstable zero cancelation and its compensation. Basically, there exists an error in the walking pattern generation method for humanoid robots using both a pole-zero cancelation by series approximation (PZCSA) and a linear quadratic regulator (LQR) method suggested in [12]. This error is caused by the approximate pole-zero cancelation for removing an unstable zero. As an alternative of this problem, firstly, we construct an estimator for position/acceleration errors of Center of Mass (CoM) and a position of Zero Moment Point (ZMP), and then they are used for compensating the ZMP error and the CoM error using a low pass filter. Through these simple procedures, finally, we show that both the ZMP and CoM errors can be removed through numerical simulations.**

***Keywords*— Walking Pattern, Humanoid Robot, Zero Moment Point (ZMP), Center of Mass (CoM), Low Pass Filter**

1. Introduction

Recently, many researches on humanoid robots have been done, specially in field of dynamic walking control. In general, three kinds of walking pattern generation methods have been reported; using a Central Pattern Generator (CPG) in [1,2], exploiting full dynamics for walking pattern generation in [2,3], and making use of a simplified dynamics such as either an inverted pendulum model in [4-10] or a rolling sphere model in [11]. Among them, since the simplified modeling of humanoid robot has been widely used for walking/posture control as well as walking pattern generation, this paper will also use it. The conventional walking pattern generation method using both a pole-zero cancelation by series approximation (PZCSA) and a linear quadratic regulator (LQR) method suggested in [12] has rooted in the preview control suggested in [4]. Though the preview method as well as the PZCSA method make use of future information of reference Zero Moment Point (ZMP) for generating the desired Center of Mass (CoM) and desired ZMP trajectories, it is said in [14] that the PZCSA method is more effective than the preview method. However, the PZCSA has necessarily error between the desired ZMP and reference ZMP because the unstable pole/zero cancelation is approximately accomplished in terms of series expansion of unstable zero [12]. Actually, according as the order of series approximation increases, the error between the desired ZMP and reference ZMP is reduced. However, increasing the order of series approximation requires much future information of reference ZMP, in other words, there is a tradeoff between the exactness reducing the error and causality requiring future information in the conventional PZCSA method. The goal of this paper is to remedy the PZCSA method for improving the exactness without increasing the order of series approximation, in other words, for removing the error without requiring much future information of ZMP. To do this, firstly, we construct an estimator for position/acceleration errors of the CoM and the ZMP, and then they are used for compensating the ZMP error and the CoM error using a low pass filter. This paper is organized as follows; section II introduces problem statement to be solved in this paper, section III proposes the compensation method of ZMP error and CoM error; section IV shows the effectiveness of the suggested method by simulations; and section V draws the conclusion.

2. Problem Statement

For the walking pattern generation, we will make use of the simplified dynamics represented by the differential equation between the ZMP and the CoM as follows:

$$p_i(t) = c_i(t) - \frac{1}{w_n^2}\ddot{c}_i(t), \qquad \text{for} \quad i = x, y, \tag{1}$$

where w_n is defined by $\sqrt{g/h}$, h is the height constant of CoM, g is the gravitational acceleration constant, c_i denotes i-directional position of CoM for $i = x, y$ and p_i denotes i-directional ZMP for $i = x, y$.

Also, we will introduce the walking pattern generation using PZCSA and LQR method suggested in [12]. This conventional method generates the desired ZMP and CoM trajectories from only the reference ZMP trajectories which are given according to desired footprints of humanoid robot. Firstly, let us define the discrete-time state vector as follows:

$$x_i[k] \triangleq \begin{bmatrix} c_{d,i}[k] \\ \dot{c}_{d,i}[k] \\ \ddot{c}_{d,i}[k] \end{bmatrix} \tag{2}$$

where $c_{d,i}[k], \dot{c}_{d,i}[k], \ddot{c}_{d,i}[k]$ denote the desired CoM position, velocity and acceleration at k-th sample for $i = x, y$, respectively. Here, we assume that the LQR controller is designed

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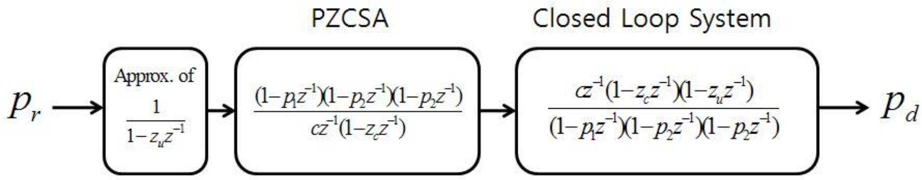

Fig. 1. Conventional method using PZCSA and LQR suggested in [12], where p_r denotes the reference ZMP, p_d the desired ZMP, z_u the unstable zero, z_c the stale zero, p_1, p_2, p_3 three stable poles

at the level of jerk (derivative of acceleration), then the discrete-time model of Eq. (1) is obtained with sampling period T as follows:

$$x_i[k+1] = Ax_i[k] + Bu_i[k] \tag{3}$$

where u_i is the typical LQR control input to be designed, A is the system matrix and B is the input distribution matrix as following forms:

$$A = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix}.$$

The design method of typical discrete-time LQR control input was suggested in detail in [12]. After the LQR control input being designed, if we choose the output vector as follows:

$$y_i[k] \triangleq \begin{bmatrix} p_{d,i}[k] \\ c_{d,i}[k] \\ \ddot{c}_{d,i}[k] \end{bmatrix} = Cx_i[k] \tag{4}$$

where $p_{d,i}[k]$ denotes the desired ZMP at the k-th sample for $i = x, y$ and C denotes the output matrix as following form:

$$C = \begin{bmatrix} 1 & 0 & -1/w_n^2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Actually, after applying the LQR to the system, if we take the transfer function from the reference to the desired ZMP, then the closed-loop control system has three stable poles and two zeros; one of both is stable zero and another is unstable zero. Now let us cancel three stable poles and one stable zero (p_1, p_2, p_3 and z_c in Fig. 1) out by applying the inverse transfer function to the closed-loop control system as shown in Fig. 1. Here, the last remaining problem is stably to realize unstable zero cancelation because unstable pole/zero cancelation gives rise to instability problem for the total system. The PZCSA suggested in [12] makes use of Taylor series expansion for following function:

$$\frac{1}{1-z_u z^{-1}} = -\frac{1}{z_u}z - \frac{1}{z_u^2}z^2 - \frac{1}{z_u^3}z^3 - \cdots$$
$$= -\sum_{n=1}^{\infty} \frac{1}{z_u^n} z^n \tag{5}$$

for $|z| < |z_u|$. Actually, exploiting this series expansion for pole/zero cancelation brings two problems, the one is a causality and the other is a realization problem to infinite series. For practical use, if we take a finite series (from 1 to m) for realizing Eq. (5), then the error is inevitably caused as follows:

$$\text{error}: \frac{1}{1-z_u z^{-1}} - \left(-\sum_{n=1}^{m} \frac{1}{z_u^n} z^n\right) = -\sum_{n=m+1}^{\infty} \frac{1}{z_u^n} z^n \tag{6}$$

where m is the order of series approximation. Till now, we have stated the problem in the walking pattern generation using PZCSA method suggested in [12]. Actually, increasing the order is able to reduce the error as we can see in Eq. (6), though it requires much future information of reference ZMP. Also, requiring much future information of reference ZMP prevents the fast walking pattern change from the abrupt environmental changes. So, we will propose the compensation method of ZMP error and CoM error in the following section.

3. Compensation Method using Low Pass Filter

In this section, we will propose a compensation method of ZMP error and CoM error as shown in Fig. 2. Firstly, the ZMP error is defined as following form:

$$e_i[k] = p_{r,i}[k] - p_{d,i}[k] \tag{7}$$

where $e_i[k]$ denotes the ZMP error between the reference ZMP $p_{r,i}[k]$ and the desired ZMP $p_{d,i}[k]$ for $i = x, y$. Secondly, the CoM error is estimated from the ZMP error by using the discrete-time low pass filter as following form:

$$\frac{E_{l,i}(z)}{E_i(z)} = \frac{T(1+z^{-1})}{(T+2\sigma)+(T-2\sigma)z^{-1}} \tag{8}$$

where $e_{l,i}[k]$ denotes the CoM error for $i = x, y$. Thirdly, the CoM acceleration error is also estimated from the following relation:

$$\ddot{e}_{l,i}[k] = w_n^2(e_i[k] - e_{l,i}[k]) \tag{9}$$

where $\ddot{e}_{l,i}[k]$ denotes the CoM acceleration error for $i = x, y$. Finally, the compensated desired ZMP and CoM are obtained as follows:

$$\hat{c}_{d,i}[k] = c_{d,i}[k] + e_{l,i}[k] \tag{10}$$
$$\hat{p}_{d,i}[k] = \hat{c}_{d,i}[k] - \frac{1}{w_n^2}(\ddot{c}_{d,i}[k] + \ddot{e}_{l,i}[k]) \tag{11}$$

where $\hat{c}_{d,i}[k]$ denotes a compensated desired CoM position and $\hat{p}_{d,i}[k]$ is a compensated desired ZMP, for $i = x, y$. Also, if the compensated desired CoM is applied to the simplified dynamics of Eq. (1), then we can confirm that the

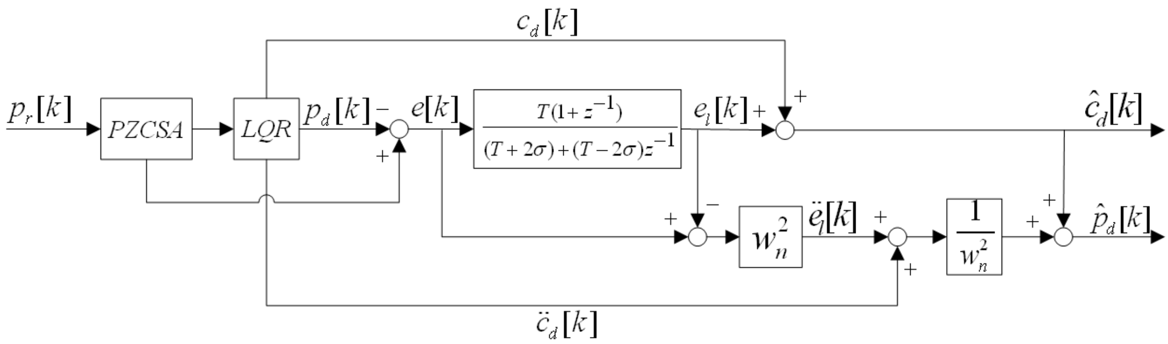

Fig. 2. Compensation scheme of ZMP error and CoM error

compensated desired ZMP is equal to the reference ZMP as follows:

$$\therefore \quad \hat{c}_{d,i}[k] - \frac{1}{w_n^2}\ddot{\hat{c}}_{d,i}[k] = \hat{p}_{d,i}[k] = p_{r,i}[k]\,.$$

As a matter of fact, the suggested compensation method was reconstructed in order to satisfy above property.

4. Numerical Simualtion

Firstly, after applying the same parameters conditions suggested in [12] as following forms:

$$J_i = \sum_{j=k}^{\infty}(p_{r,i}[j] - p_{d,i}[j])^2 + 10^{-6}(u_i[j] - u_i[j-1])^2$$
$$T = 0.005[s]$$
$$g = 9.8[m^2/s]$$
$$z_c = 0.814[m]$$
$$m = 320[future\ samples]$$

to the conventional PZCSA method for 500 seconds, we could obtain the simulation result as shown in Fig. 3. This walking pattern generation method using conventional PZCSA method seems to be good, however, if we investigate closely small regions about 350 seconds, then we can see the error between the reference ZMP and desired ZMP as shown in Fig. 4.

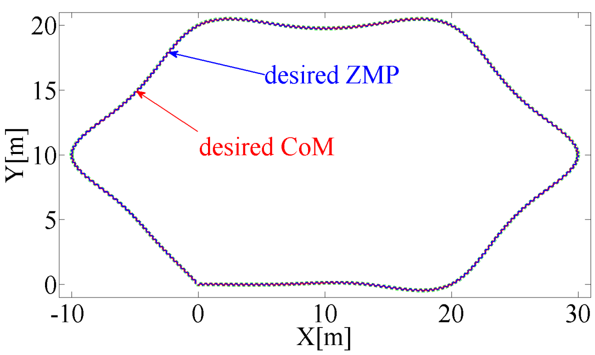

Fig. 3. Simulation result of PZCSA method in [12] for 500 seconds

This errors shown in Fig. 4 were caused by the series approximation with $m = 320[samples]$ future values. Just in

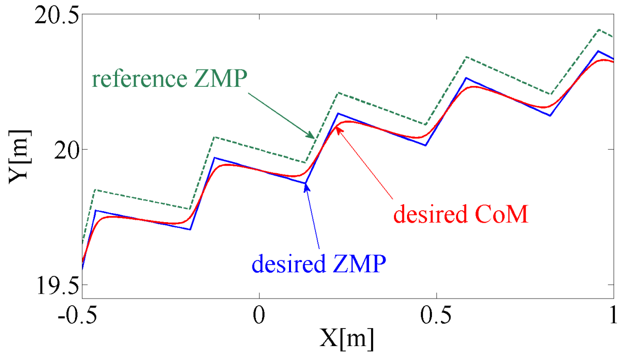

Fig. 4. Small region of Fig. 3 about 350 seconds

cases, if we apply the suggested compensation method to the conventional PZCSA one as shown in Fig. 2 with the filter time constant of low pass filter as follow:

$$\sigma = 0.0831\,,$$

then we can remove the ZMP error and CoM error as shown in Fig. 5.

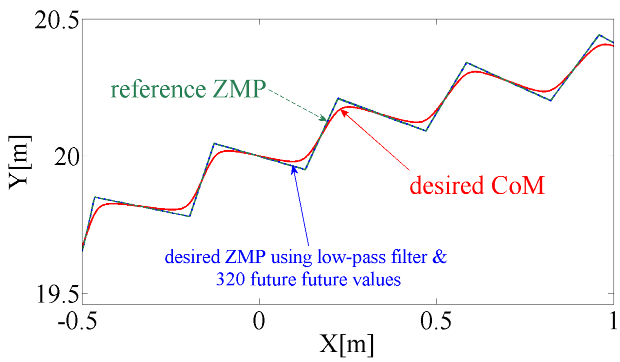

Fig. 5. Simulation results using the suggested compensation method

5. Concluding Remarks

In this paper, the defect of walking pattern generation method using an approximate unstable zero cancelation was shown through the ZMP and CoM error analysis. To reduce the error, the compensation method was suggested in this paper. Finally, the effectiveness of the suggested method was shown through the numerical simulations.

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References

[1] W. Yang, N. Chong, C. Kim, and B. You, "Self-adapting Humanoid Locomotion Using a Neural Oscillator Network" Proc. of IEEE Int. Conf. on Intelligent Robotics and Systems, PP. 309-316, 2007.
[2] K. Hirai, M. Hirose, Y. Haikawa and T. Takenaka, "Development of Honda Humanoid Robot," Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 1321-1326, 1998.
[3] J. Yamaguchi, E. Soga, S. Inoue and A. Taknishi, "Development of a Bipedal Humanoid Robot -control Method of Whole Body Cooperative Dynamic Biped Walking-," Proc. Of IEEE Int. Conf. on Robotics and Automation, pp. 2299-2306, 1999.
[4] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi and H. Hirukawa, "Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point," Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 1620-1626, 2003.
[5] Y. Oh, K. Ahn, D. Kim and C. Kim, "An Analytical Method to Generate Walking Pattern of Humanoid Robot," Pro. Of IEEE Int. Conf. on Industrial Electronics Society, pp. 4159-4164, 2006.
[6] K. Harada, S. Kajita, K. Kaneko and H. Hirukawa, "An Analytical Method on Real-time Gait Planning for a Humanoid Robot," IEEE RAS/RSJ Int. Conf. on Humanoid Robots, pp. 640-655, 2004.
[7] Q. Huang, K. Yokoi, S. Kajita, K. Kaneko, H. Arai, N. Koyachi and K. Tani, "Panning Walking Patterns for a Biped Robot," IEEE Trans. On Robotics and Automation, pp. 280-289, 2001.
[8] C. Zhu, Y. Tomizawa, X. Luo and A. Kawamura, "Biped Walking with Varible ZMP, Fricitional Constraint and Inverted Pendulum Model", Proc. of IEEE Int. Conf on Robotics and Biomimetics, pp. 425-430, 2004.
[9] K. Loffler, M. Gienger, and F. Pfeiffer, "Sensor and Control Design of a Dynamically Stable Biped Robot," Proc. of IEEE Int. Conf. on Robotics and Automation, pp.484-490, 2003
[10] S. Hong, Y. Oh, Y. Chang, and B. You, "An omni-directional Walking Pattern Generation Mehtod for Humanoid Robots with Quartic Polynomials" Proc. of IEEE Int. Conf. on Intelligent Robotics and Systems, pp.4297-4213, 2007.
[11] Y. Choi, D. Kim, and B. You, "On the walking control for humanoid robot based on the kinematic resolution of CoM Jacobian with embedded motion ," Proc. of IEEE Int. Conf. on Robotics and Automation, pp.2655-2660, 2006.
[12] S. Hong, Y. Oh, Y.-H. Chang, B.-J. You, "Walking pattern generation for Humanoid robots with LQR and feedforward control method," IEEE Int. Conf. on Industrial Electronics, pp.1698-1703, 2008
[13] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi and H. Hirukawa, "Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point," Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 1620-1626, 2003.
[14] E. Gross, M. Tomizuka,W. Messner, "Cancellation of Discrete Time Unstable Zeros by Feedforward Control," Journal of Dynamic Sysmtems, Measurement, and Control, Vol. 116 ,pp. 33-38, 1994.