We investigate the structure of a proto-neutron star (PNS) with trapped neutrinos by using quantum hadrodynamics (QHD). Ratios of the trapped neutrinos and corresponding leptons to the baryons are usually assumed as a constant. But relevant reactions associated to the beta equilibrium could be sensitive to the given density and temperature. By adopting a phenomenological lepton density which is smeared near the surface, we calculate and discuss populations of baryons and leptons, equations of state and the mass-radius relation of a PNS in isentropic process ($S = 2$).

Calculation for a neutron star is usually constrained by three conditions, baryon number conservation, charge neutrality and beta equilibrium. But for a PNS with trapped neutrinos, the beta equilibrium has to be modified with trapped neutrinos $\mu_\nu$,

$$\mu_\nu = \mu_p + \mu_e - \mu_\nu. \quad (1)$$

In the beta equilibrium, the matter becomes symmetric matter if $\mu_e = \mu_\nu$. By considering that the neutrino propagation and the beta equilibrium may depend on density and temperature, density of electron neutrinos, $\rho_\nu e$, is assumed to be related as $\rho_\nu e = x(\rho)\rho_e$. The condition for muon production is also taken as $\rho_\nu \mu = x(\rho)\rho_\mu$ by satisfying the chemical equilibrium $\mu_e + \mu_\nu = \mu_\mu + \mu_\nu$. Here $x(\rho)$ may depend on baryon density and temperature. However, to make the problem simple, we assume that $x(\rho)$ depends on baryon density by using a phenomenological formula,

$$x(\rho) = x_0 \left[ 1 - \exp(-\beta (\rho/\rho_0)^\gamma) \right]. \quad (2)$$

where $\beta = 0.05$ and $\gamma = 2$ are used. When we ignore the mass of a electron, $x(\rho)$ is nearly $x(\rho) \approx 0.5$ for $\mu_e = \mu_\nu$ because the degeneracy factor of neutrino is 1. Since the condition, $x(\rho) > 0.5$, means proton rich matter, we take the $x(\rho)$ as $x < 0.5$ in all regions of a PNS. Thus, in this work, we use $x_0 = 0.3$. This ratio function implies smooth smearing of relevant leptons at the surface of a PNS.

With these formula and conditions, we obtained populations of neutrinos in Fig. 1. When comparing our results with the simple fixed lepton model [1], the ratio of neutrinos from our formula agrees with those from the simulation of transport theory [2,3]. We thus think our assumptions can explain the results of the simulation and give a physical insight for a PNS. Although our model does not have time evolution, it suggests a phenomenological model about the trapped neutrino which can be or might be reproduced by the realistic dynamical model.