PHYSICAL REVIEW C **83**, 025206 (2011)

Half-Skyrmions, tensor forces, and symmetry energy in cold dense matter

Hyun Kyu Lee,[1] Byung-Yoon Park,[2] and Mannque Rho[3,4]
[1]*Department of Physics, Hanyang University, Seoul 133-791, Korea*
[2]*Department of Physics, Chungnam National University, Daejon 305-764, Korea*
[3]*Institut de Physique Théorique, CEA Saclay, F-91191 Gif-sur-Yvette Cédex, France,*
[4]*Department of Physics, Hanyang University, Seoul 133-791, Korea*
(Received 30 November 2010; published 18 February 2011)

In a previous article, the four-dimensional (4D) half-Skyrmion (or five-dimensional dyonic salt) structure of dense baryonic matter described in crystalline configuration in the large N_c limit was shown to have nontrivial consequences on how antikaons behave in compressed nuclear matter with a possible implication for the "ice-9" phenomenon of deeply bound kaonic matter and condensed kaons in compact stars. We extend the analysis to make a further prediction on the scaling properties of hadrons that have a surprising effect on the nuclear tensor forces, the symmetry energy, and hence on the phase structure at high density. We treat this problem, relying on certain topological structures of chiral solitons. Combined with what can be deduced from hidden local symmetry for hadrons in a dense medium and the "soft" dilatonic degree of freedom associated with the trace anomaly of QCD, we uncover a novel structure of chiral symmetry in the "supersoft" symmetry energy that can influence the structure of neutron stars.

DOI: 10.1103/PhysRevC.83.025206 PACS number(s): 21.65.Ef, 26.60.−c, 11.30.Rd, 12.39.Dc

I. INTRODUCTION

When A Skyrmions with $A \to \infty$ are put on a face-centered cubic (fcc) crystal lattice and squeezed to simulate dense baryonic matter, the Skyrmion in the system is found to fractionize into two half-Skyrmions at a density $n_{1/2} \sim x n_0$ with $x > 1$ where n_0 is the normal nuclear-matter density [1]. The matter made up of the half-Skyrmions is characterized by the vanishing quark condensate $\langle \bar{q}q \rangle \propto \mathrm{Tr}U = 0$ and the nonvanishing pion decay constant $f_\pi \neq 0$, whereas the lower-density Skyrmion state has both $\langle \bar{q}q \rangle \neq 0$ and $f_\pi \neq 0$, symptomatic of chiral symmetry spontaneous breaking. A similar structure was found [2] with instantons on an fcc crystal fractionizing into two half-instantons (or dyons) in five-dimensional (5D) Yang-Mills theory in the gravity sector that arises in holographic QCD [3]. What distinguishes the instanton baryon in the bulk gravity sector from the Skyrmion baryon in the boundary gauge sector is that the former involves an infinite tower of vector mesons, so the physics of highly dense matter will be more efficiently accessed with higher energy degrees of freedom incorporated.

It was found in Ref. [1] that the mass of an antikaon propagating in dense matter undergoes a more propitious decrease as Skyrmions fractionize into half-Skrymions. It was suggested that this behavior could trigger a deeply bound kaonic state in nuclear matter, a sort of "ice-9" phenomenon,[1] and kaon condensation in neutron star matter at a lower density than thought previously. In this paper, based on certain generic structure of the dense Skrymion matter, we make several further predictions, specifically on the structure of the tensor forces operative in dense matter at $n > n_0$ and its ramifications on the symmetry energy of asymmetric nuclear matter and consequently on the structure of compact stars.

II. THE MODEL

The model we use to study cold dense baryonic matter put in crystals is the two-flavor Skyrme model with the quartic Skyrme term, supplemented with a dilaton scalar χ, analyzed in [1],

$$\mathcal{L}_{\rm Sk} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{Tr}(L_\mu, L_\nu)^2 + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^3 \mathrm{Tr}\mathcal{M}(U + U^\dagger - 2) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + V(\chi), \tag{1}$$

where $V(\chi)$ is the potential that encodes the trace anomaly involving the soft dilaton (precisely defined in [4]), $L_\mu = U^\dagger \partial_\mu U$, with U the chiral field taking values in SU(2), and f_χ is the vev of χ.

In extracting the information needed for describing hadrons, both bosonic and baryonic, in a dense medium, we consider the Lagrangian (1)—without the scalar field—as "gauge equivalent" to the hidden local symmetry (HLS) Lagrangian with the U(2) multiplet ρ and ω [5]. More precisely, one can think of Eq. (1) as resulting from integrating out all vector mesons from a Lagrangian that contains an infinite tower of hidden local fields, such as in the holographic QCD (hQCD) model of Sakai and Sugimoto [3] based on string theory or deconstructed bottom up from the low-energy current algebra term [6]. Thus, although we simulate the Skyrmion matter with Eq. (1), we are able to make statements on the vector mesons, which are not explicit degrees of freedom in the Lagrangian. While Eq. (1) has been widely studied both for the elementary

[1]It was first pointed out to one of us (M.R.) by Gerry Brown that this phenonomenon is somewhat like the "Ice-9" in K. Vonnegut's novel *Cat's Craddle* (Holt, Rinehardt & Winston, New York, 1963).

nucleon and for many-nucleon systems in the literature, we find that it has a surprising feature that has so far remained unexposed, particularly in many-nucleon systems. It makes certain predictions based on topological structure of the soliton contained in the model, which has some dramatic effects on compressed baryonic matter relevant to compact stars.

Oversimplified as it may appear, the Lagrangian (1) could be justified as a candidate effective field theory for the physics of dense matter on several grounds. The first term without the coupling to the dialton field is of course the current algebra term rigorously valid at very low energy. The second term, called the Skyrme term, often considered as *ad hoc*, is also justifiable. Although it is in principle not the only term that can appear in the chiral Lagrangian at fourth order in derivative, chiral perturbation calculations of π-π scattering show that it is the dominant term, with other terms essentially canceling out [7]. This contrasts with the linear σ model in which the three other four-derivative terms combined together are found to destabilize the soliton and make it collapse to a point. Other information on the Skyrme term comes from hQCD constructed by Sakai and Sugimoto [3] in which, present with the infinite tower of vector mesons, it turns out to be the only term quartic in derivatives acting on the pion field. In fact, the coefficient $1/e^2$ in the hQCD Lagrangian is precisely fixed by $\frac{1}{216\pi^2}\lambda N_c$ where λ is the 't Hooft constant and N_c is the number of colors. Surprisingly, this coefficient comes out numerically very close to what has been found in the Skyrme model.[2]

More generally, as mentioned previously, one may consider the Skyrme model as resulting from integrating out *all* vector degrees of freedom from the infinite tower of hidden local gauge fields that arises as emergent or "deconstructed" bottom up from the current algebra or reduced top down from the 5D YM theory in hQCD.[3] The Skyrme term may be taken to encapsulate short-distance degrees of freedom that include quarks and gluons. Of course, there is no reason why one can simply stop at the quartic order in derivative. In general, with higher order terms, the number of parameters increases rapidly, although within the holographic model à la Sakai-Sugimoto, the situation is somewhat ameliorated.

In using Eq. (1), we choose to pick, as advocated in [11], the parameters of the Lagrangian determined in the meson sector, not taken as free parameters as has been usually done in the literature. We thus take the pion decay constant to be given by $f_\pi \approx 93$ MeV and the Skyrme term constant $1/e^2$ as given by the lowest mass scale integrated out, namely, the vector meson mass, $m_\rho \approx \sqrt{2} f_\pi e$. Of course, the nucleon mass comes out too high with these constants, say, $\sim$1500 MeV, but this is the mass given at the leading order, $\mathcal{O}(N_c)$. As such, this high value should not worry us. In fact, the next order $[\mathcal{O}(N_c^0)]$ term, that is, the Casimir term (which is difficult to calculate precisely)- is estimated to be approximately -500 MeV.[4]

In applying the Skyrme Lagrangian to many-nucleon systems, one glaring defect is the missing scalar degree of freedom that plays a key role in nuclear dynamics. This was recognized already in 1991 when the scaling relations were first written down [12]. Unlike the scalar σ in the linear σ model, which does not support stable nuclear matter, the χ field in Eq. (1) is a chiral scalar locked to the chiral condensate $\langle\bar{q}q\rangle$. How to introduce the scalar field χ in chiral Lagrangians—which is not at all trivial—was discussed in [4]. What is needed for our purpose is the "soft dilaton" figuring in the trace anomaly of QCD whose condensate is locked to the chiral condensate. This scalar mode can be thought of as representing the vibrational mode (i.e., the Casimir effect)—which is subleading in N_c—missing in the Skyrme model mentioned previously. This interpretation is supported by the result obtained in [13], where it is found that for large dilaton mass $m_\chi \gtrsim$ GeV, the soliton mass $M_{\rm sol}$ comes out to be $\gtrsim$ GeV, whereas for $m_\chi < 1$ GeV, it is $\gtrsim$1 GeV. Thus, the role of the soft dilaton is equivalent to that of the $\mathcal{O}(N_c^0)$ Casimir effect. Furthermore, multiplying the Wess-Zumino term $\propto \omega B$ with χ^n with $n \gtrsim 2$ also simulates *by fiat* the property of "vector manifestation" of hidden local symmetry that dictates that the vector meson coupling to the pions vanishes $\propto \langle\bar{q}q\rangle$ as the chiral transition point is approached in the chiral limit [9]. This property could be—and should be—more efficiently addressed in the framework of hidden local symmetry Lagrangian with the hidden local fields present together with the pions and the dilaton. By putting the ω meson together with the ρ meson in an U(2) multiplet, it can counterbalance the possible overbinding by the scalar χ field as discussed later.[5]

[2] The hQCD model [3] gives $1/e^2 \approx 2.51\frac{\lambda N_c}{216\pi^3} \approx 0.02$ for $N_c = 3$ and $\lambda \approx 17$, which is fit by the meson and baryon properties. This is comparable to what one obtains using the mass formula $m_\rho^2 = 2 f_\pi^2 g_V^2 \approx 770$ MeV, i.e., $1/g_V^2 \approx 0.03$.

[3] It has recently been shown [8] that if all vector mesons *except* for the lowest $[V_0 = (\rho,\omega)]$ are integrated out in a way consistent with hidden local symmetry for *all* vector mesons in the Sakai-Sugimoto model [3], the resulting Lagrangian for V_0 (with the Goldstone π) is precisely the HLS theory proposed in [5,9] with the parameters of the Lagrangian fixed by the 5D holographic QCD. This Lagrangian is found to give a new interpretation of vector dominance for both the pion [8] and the nucleon [10] in their electromagnetic form factors. It is also interesting to note that the Skyrme Lagrangian with only the current algebra term and the Skyrme quartic term is obtained (in the bulk sector) when *all* vector mesons are integrated out in the hidden local symmetric way. This can be interpreted as the statement (in the gauge sector) of "gauge equivalence" between the nonlinear σ model and HLS theory [5,9].

[4] The N_c counting goes as follows: $\sim$1500 MeV at $\mathcal{O}(N_c)$, approximately -500 MeV at $\mathcal{O}(N_c^0)$, and $\sim$300 MeV at $\mathcal{O}(1/N_c)$ (from $N-\Delta$ mass difference). We see nothing unreasonable in this counting.

[5] In the hQCD model [3], it is the U(1) field in the Chern-Simons term—and not the Skyrme quartic term—that stabilizes the instanton. It is possible that this role is played by the ω field also in the HLS model. This problem was addressed in [13] with an HLS Lagrangian in unitary gauge. However, the analysis made there used an approximation that is most likely invalid in dense medium: There the anomalous Lagrangian consisting of four homogeneous Wess-Zumino (hWZ) terms was approximated by only one term proportional to $\omega_\mu B^\mu$ where B^μ is the baryon current. The assumption there was that the vector meson stays "heavy" at any density, which

III. HALF-SKYRMION CRYSTAL

We now specify the features of the dense matter constructed with the Lagrangian (1) that are exploited in this paper.

The method we use to describe dense baryonic matter with the chiral Lagrangian (1), valid at large N_c, is to put multiple Skyrmions on crystal lattice as pioneered by Klebanov [14] and squeeze the system to simulate density. The most recent review on this approach is found in Ref. [15]. Here we rely on the results obtained in Ref. [1] with the Lagrangian (1) by putting Skyrmions on an fcc crystal. With the parameters f_π and $1/e^2$ fixed as described previously, there is only one parameter remaining to be fixed, namely, the mass of the dilaton m_χ. At present, there is no clear information on m_χ, both experimentally and theoretically. There is a great deal of controversy on scalar mesons involving both quarkonic and gluonic configurations. In the absence of better guidance, we take two values that we consider reasonable for our problem. One is ~600 MeV, which corresponds, roughly, to the lowest scalar with a broad width listed in PDG. This is the mass compatible with relativistic mean field theory of nuclear matter. The other is ~700 MeV, which figures as an effective scalar meson in the chiral Lagrangian with the parameters scaling with density [16]. Given the uncertainty, these values should be taken as simply representative. For definite results, we focus more on $m_\chi \approx 700$ MeV.

The results of Ref. [1] that are essential for what follows are as follows:

(i) The state of Skyrmions in fcc makes a phase transition to a half-Skyrmion matter in cubic crystal (cc) at $n = n_{1/2} > n_0$ (where $n_0 \approx 0.16\ \mathrm{fm}^{-3}$ is the normal nuclear-matter density). It is found to be fairly independent of the mass of the scalar χ [17]: Even at 1200 MeV, it differs negligibly from that at 700 MeV. However, it is sensitive to the Skyrme parameter e going as $\sim e^3$ and more generally to certain HLS parameters such as the HLS coupling g, so it is difficult to pin down the density $n_{1/2}$. For the given f_π and e, it comes at the range $(1.3-2)n_0$. This should be taken as representative. What is important for our purpose is that the $n_{1/2}$ be not too far above n_0. Were it to be so, then the effect of the phase change would be unimportant in the process we are concerned with. We note that in this phase, the quark condensate $\langle\bar{q}q\rangle^* \propto (\mathrm{Tr}U)^*$ vanishes *on averaging* while the average value of the amplitude field remains nonzero.

(ii) As density is increased beyond $n_{1/2}$, a phase change takes place at n_c to a matter where f_π^* drops to zero. The phase change appears to be of first order. The critical density n_c for this phase with $\langle\bar{q}q\rangle^* = f_\pi^* = 0$ turns out to be extremely sensitive to the dilaton mass. For instance, $n_c/n_0 \approx (4\text{–}25)$ for $m_\chi \approx (700\text{–}1200)$ MeV [17]. This phase could be identified as the chiral-symmetry restored and quark-deconfined phase.

is at odds with the vector manifestation (VM) property. A correct calculation with all four hWZ terms remains to be done.

IV. NEW SCALING

From these results, we infer the following consequences on in-medium scaling.

A. Scaling of the nucleon mass

Within the range of density involved, the large N_c piece of the effective (or quasi-) nucleon mass in the model scales as $m_N^* \sim f_\pi^*/e$. Since e is scale invariant, the scaling is only in f_π^*. As noted, f_π^* remains nonzero, with the ratio f_π^*/f_π dropping roughly linearly in density to a nonzero value at n_c.[6] To a good first approximation, we can simply take

$$m_N^*/m_N \approx f_\pi^*/f_\pi \qquad \text{for } 0 \gtrsim n \gtrsim n_{1/2} \tag{2}$$

$$\approx b \qquad \text{for } n_{1/2} \gtrsim n \gtrsim n_c, \tag{3}$$

where b is a constant, a reasonable range of which is $b \sim 0.6$–0.8. What we have here resembles the result obtained in the parity-doubled linear or nonlinear σ model where the chiral invariant mass m_0 comes out to be $m_0 \sim 0.5$–0.8 MeV [18].

B. Scaling of the vector-meson mass

For the properties of vector mesons, we are guided by the HLS theory to which (1) is gauge equivalent [5,9]. We expect the vector-meson mass to take the form (in the leading N_c order)

$$m_V^* \approx f_\pi^* g_V^*, \tag{4}$$

where g_V^* is the hidden gauge coupling constant and $V = \rho, \omega$. We have set $a^* = (f_\sigma^*/f_\pi^*)^2 = 1$ for large N_c. There is no theoretical argument as to how g_V^* scales in density up to n_0 (or $n_{1/2}$). However, thermal lattice calculations indicate that there is practically no scaling up to near the critical temperature. In Ref. [19], this observation was simply carried over to the density case. We were led to assume that g_V^* does not scale up to $n_{1/2}$. In Ref. [19], this argument is given some support from nuclear dynamics. However, the renormalization group argument based on hidden local symmetry theory shows that as one approaches the chiral critical point n_c, it should scale to zero proportionally to $\langle\bar{q}q\rangle$ very near the VM fixed point [9]. Thus, we infer the following scaling:

$$\frac{m_V^*}{m_V} \approx f_\pi^*/f_\pi \equiv \Phi \qquad \text{for } 0 \gtrsim n \gtrsim n_{1/2} \tag{5}$$

$$\approx b\frac{g_V^*}{g_V} \equiv b\Phi' \qquad \text{for } n_{1/2} \gtrsim n \gtrsim n_c, \tag{6}$$

where the two different scaling functions Φ and Φ' are defined. It is important to note that the vector-meson mass scales with

[6] This observation is consistent with the pion decay constant "measured" in deeply bound pionic nuclei, $f_\pi^*/f_\pi \approx 0.8$ at the nuclear matter density. In finite nuclei, there are nuclear corrections subleading in N_c that we are not considering. Note that in HLS theory [9], it is the loop corrections higher order in $1/N_c$ that drive the physical (renormalized) pion decay constant to zero at the chiral transition.

f_π^* up to $\sim n_{1/2}$ but with g_V^* for $n > n_{1/2}$. This observation was made already in Ref. [19] and applied to kaon condensation in Ref. [20]. It is the scaling (5) that is revealed in the C14 dating [21]. In hidden local symmetry theory without baryon degrees of freedom [9], g_V^* scales $\propto \langle\bar{q}q\rangle^*$, but given the presence of the half-Skyrmion phase, this scaling could be modified. It is certain that it will go to zero at the VM fixed point even in the presence of baryons.

V. EFFECT ON NUCLEAR TENSOR FORCES

We now apply the scaling relations Eqs. (2)–(6) to nuclear tensor forces. That the scaling proposed in Ref. [12] could strongly affect the nuclear tensor forces in nuclear matter has been known for some time [19]. The new scalings leave the behavior up to n_0 unchanged from the previous scaling but drastically modify the properties above the nuclear matter density.

Since the nucleon is massive in our model in the whole range of density we are dealing with, the nonrelativistic approximation is valid, so the two-body tensor forces contributed by one pion and one ρ exchange maintain the familiar form

$$V_M^T(r) = S_M \frac{f_{NM}^2}{4\pi} m_M \tau_1\cdot\tau_2 S_{12} \times \left(\left[\frac{1}{(m_M r)^3}+\frac{1}{(m_M r)^2}+\frac{1}{3m_M r}\right]e^{-m_M r}\right), \quad (7)$$

where $M=\pi,\rho$ and $S_{\rho(\pi)}=+1(-1)$. The key aspect of these forces is that there is a strong cancellation between the two. This cancellation plays the crucial role in the C12 dating problem in Ref. [21].

Thus far, we have not addressed how pion properties scale. If the pion mass were zero, they would be protected by chiral invariance and hence would remain unscaled. However, with nonzero pion mass, the situation could be different. Since the chiral symmetry is only lightly broken, the in-medium property of the pion is subtle and requires an extremely careful treatment. Such an analysis by Jido, Hatsuda, and Kunihiro yielded the in-medium Gell-Mann-Oakes-Renner relation [22]

$$m_\pi^*(n)/m_\pi \approx \left[f_\pi^t(n)/f_\pi\right]^{-1}(\langle\bar{q}q\rangle^*(n)/\langle\bar{q}q\rangle)^{1/2}, \quad (8)$$

where f_π^t is the time component of the pion decay constant, which differs from the space component in medium. Using the experimental information available at the nuclear matter density [23], $[f_\pi^t(n_0)/f_\pi]^2 \simeq 0.64$ and $\langle\bar{q}q\rangle^*(n_0)/\langle\bar{q}q\rangle \simeq 0.63$, we get $m_\pi^*/m_\pi \simeq 1$, so there is no noticeable change in the pion mass up to the nuclear matter density. It seems reasonable to assume that up to the density we are concerned with—which is not too far above n_0—the pion properties remain unscaled. This should be good enough for our discussion. Taking into account the small pion mass effect in a more precise way would require a highly detailed treatment that the model used here is not equipped to handle and that is not warranted for the qualitative aspect we are exploring in this paper.

To see how the ρ tensor force scales, we need to see how the strength $f_{N\rho}$ scales. Plugging in the vector (or hidden gauge) coupling g_V, the strength scales as

$$R \equiv \frac{f_{N\rho}^*}{f_{N\rho}} \approx \frac{g_V^*}{g_V}\frac{m_\rho^*}{m_\rho}\frac{m_N}{m_N^*}. \quad (9)$$

It follows from the scaling relations (2)–(6) that

$$R \approx 1 \quad \text{for} \quad 0 \gtrsim n \gtrsim n_{1/2} \quad (10)$$

$$\approx \Phi'^2 \quad \text{for} \quad n_{1/2} \gtrsim n \gtrsim n_c. \quad (11)$$

In Ref. [21], Eqs. (5) and (10) were used to explain the long lifetime in the C14 dating β decay. The change of scaling that takes place at $n > n_{1/2}$ was not probed in that process. This plays the key role in the suppression of the Gamow-Teller matrix element that enters in the process. Were one to extend the scalings (5) and (10) employed in Ref. [21] to higher densities, one would find that the net tensor force would be nearly completely suppressed for the internucleon separation $r \gtrsim 1.5$ fm [24] at $n \sim 3n_0$. However, this behavior is drastically modified for $n \gtrsim n_{1/2}$ by the change of the scaling (11). Because of the strong quenching of the ρ tensor strength while the mass drops, the cancellation between the two tensor forces gets abruptly weakened as density passes $n_{1/2}$. In fact, with Φ' (assumed to be) scaling linearly in density, the ρ tensor force gets more or less completely killed at $n \gtrsim 2n_0$, leaving only the π tensor operative, in a stark contrast to the naive scaling that suppresses the total tensor instead.

To give a quantitative idea of the drastic change that takes place due to the half-Skyrmion phase, we compare the behavior of the tensor forces as a function of density between the "old" and "new" scalings. They are given in Figs. 1 and 2 for the choices of the scalings indicated therein. For simplicity of illustration, we took $b \approx 1$ and $\Phi = \Phi' \approx 1 - 0.15n/n_0$. Given that the scalings Φ and Φ' are unknown for $n > n_0$, what we have gotten here is qualitative at best. Numerically the results are not sensitive to the value of b near 1, but they could depend quantitatively on the way that Φ and Φ' scale.

VI. SYMMETRY ENERGY

The strong suppression of the ρ tensor will clearly have a big effect on the structure of baryonic matter at high density.

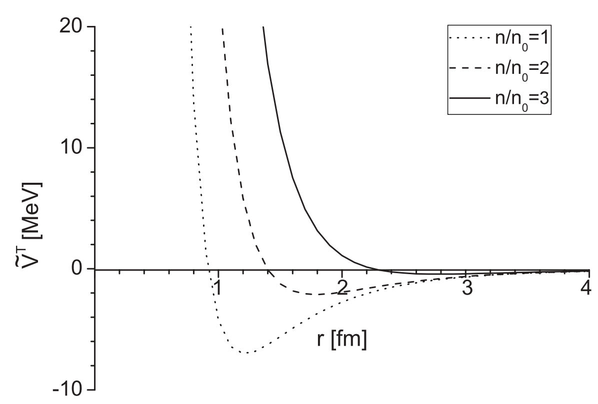

FIG. 1. Sum of π and ρ tensor forces $\tilde{V}^T \equiv (\tau_1\cdot\tau_2 S_{12})^{-1}(V_\pi^T + V_\rho^T)$ in units of million electron volts for densities $n/n_0 = 1$, 2, and 3 with the "old scaling," $\Phi \approx 1 - 0.15n/n_0$ and $R \approx 1$ for all n.

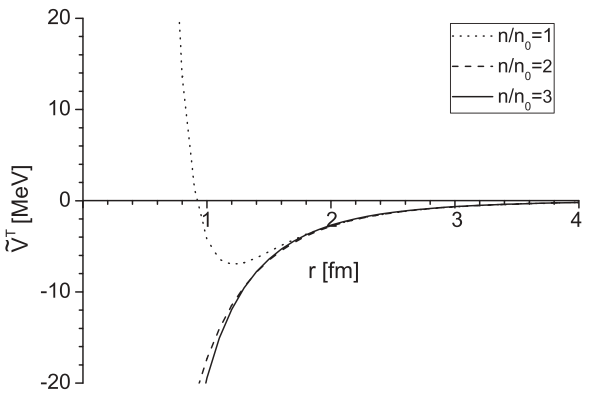

FIG. 2. The same Fig. 1 with the "new scaling," $\Phi \approx 1 - 0.15n/n_0$ with $R \approx 1$ for $n < n_{1/2}$ and $R \approx \Phi^2$ for $n > n_{1/2}$, assuming $n_0 < n_{1/2} < 2n_0$.

We test this feature in the nuclear symmetry energy that figures importantly in the structure of neutron-rich nuclei and more crucially in neutron stars. The most dramatic effect can be illustrated with the "supersoft" symmetry energy recently discussed in Refs. [24–26].

The energy per particle of asymmetric nuclear matter is given by

$$E(n,\delta) = E_0(n) + E_{\rm sym}(n)\delta^2 + \cdots, \quad (12)$$

where $\delta = (N - P)/(N + P)$ with $N(P)$ the number of neutrons (protons) and the ellipsis stands for higher orders in δ. We focus on the "symmetry energy" $E_{\rm sym}$. Fit to experimental data up to n_0, most of the symmetry energy predicted theoretically are found to increase monotonically up to n_0 with, however, a wide variation above n_0 due to the paucity of experimental constraints and the lack of reliable theory. In Ref. [25], what we might refer to as a "nonstandard form" of $E_{\rm sym}$ that increases up to and turns over at $\sim n_0$, deviating from the standard form and vanishing near $3n_0$, is argued to be required by the FOPI/GSI data on π^-/π^+ data. The schematic form of such $E_{\rm sym}$ is shown as "supersoft" in Fig. 3. It is immediately clear that such a nonstandard symmetry energy will have dramatic consequences in nuclear physics, astrophysics, and other areas. For instance, it would modify the Newtonian gravity [26], falsify the scenario of kaon condensation in compact-star formation and collapse to black holes [27], and so forth. Whether or not such a supersoft $E_{\rm sym}$ (SSE for short) is picked by nature will be tested in forthcoming experiments at a variety of laboratories such as RIB and FAIR/GSI. Leaving that issue to the future, let us take the SSE as an illustration of an extreme case and ask whether and how our scaling enters into the structure of that symmetry energy. In fact, we were motivated to ask this question by the work of Xu and Li [24], who have shown that if the tensor forces with the scaling (5) and (10) *applied to all densities* (or alternatively three-body forces) were taken into account, then the "standard symmetry energy" that increases in density continuously could be made to turn over at $\sim n_0$ and take the form of the SSE.

The key mechanism exploited in Ref. [24] is the cancellation taking place *at and beyond* $n \gtrsim n_0$ in the tensor

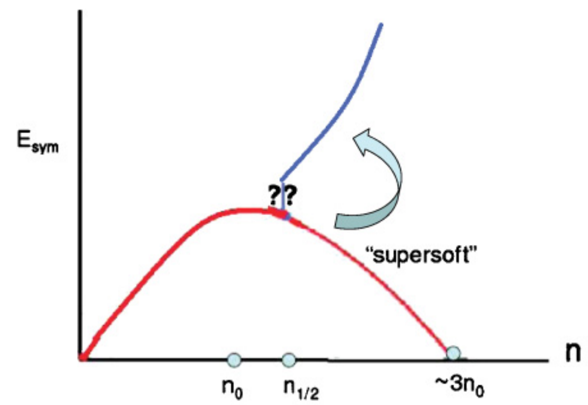

FIG. 3. (Color online) A cartoon of the symmetry energy. The scaling operative in the half-Skyrmion phase brings the abrupt change from attraction to repulsion as indicated by the arrow. The changeover region indicated by ?? is not understood, as described in Fig. 2 of [19].

forces. Our prediction, in contrast, is that the cancellation in question will cease precisely when the Skyrmion fractionizes into half-Skyrmions at $n_{1/2}$. Given the abrupt suppression of the ρ tensor, we expect that the curve will turn over from decreasing to increasing at $n_{1/2}$. Our expectation based on the new scaling is schematically shown in a cartoon in Fig. 3. As stressed in Ref. [19], it is not known how the changeover takes place. It may or may not have a discontinuity, but our analysis suggests that the slopes before and after $n_{1/2}$ will differ.

VII. SYMMETRY ENERGY IN HALF-SKYRMION MATTER

In order to make Fig. 3 realistic so as to apply to neutron-star systems, one would have to formulate a microscopic approach to implement the tensor forces with the predicted scaling in both E_0 and $E_{\rm sym}$. It may require implementing three-body forces as well. It will be a highly involved calculation that would require a lot more work; it is being pursued at present [28].

Here we suggest that the new scaling can be "seen" directly from the half-Skyrmion matter on which our scaling relations are based. In the Skyrmion framework, the symmetry energy comes from a term subleading in N_c. It must therefore arise from the collective quantization of multi-Skyrmion systems. In Ref. [29], the Skyrme model (supplemented with a six-derivative term but without the dilaton field) was collective-quantized to obtain the Weizsäcker-Bethe-Bacher formula for neutron-rich (even and odd A) nuclei from $A = 6$ to $A = 32$. The symmetry energy so obtained is in good agreement with experimental spectra. This suggests using the same technique to compute the symmetry energy from the Skyrmion crystal. In his original work on the Skyrmion crystal, Klebanov [14] discussed how to collective-quantize the pure neutron system. We apply this method to the Skyrmion matter as well as to the half-Skyrmion matter we have obtained.

Consider an A-nucleon system for $A \to \infty$. Following Klebanov, the whole matter is rotated through a single set of

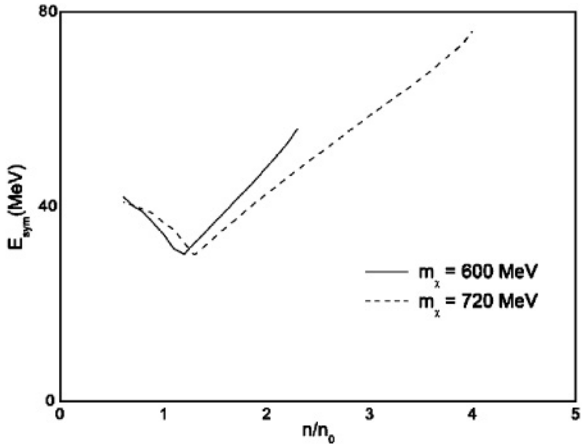

FIG. 4. Symmetry energy given by the collective rotation of the Skyrmion matter with $f_\pi = 93$ MeV, $1/e^2 \approx 0.03$, and two values of dilaton mass. The cusp is located at $n_{1/2}$. The low-density part that cannot be located precisely is not shown as the collective quantization method used is not applicable in that region.

collective coordinates $U(\vec{r},t) = A(t)U_0(\vec{r})A^\dagger(t)$, where $U_0(\vec{r})$ is the static crystal configuration with the lowest energy for a given density. The canonical quantization leads to

$$E^{\rm tot} = AM_{\rm cl} + \frac{1}{2A\lambda_I} I^{\rm tot}(I^{\rm tot}+1), \tag{13}$$

where $M_{\rm cl}$ and λ_I are, respectively, the mass and the moment of inertia of the single cell. The moment of inertia is of the form

$$\begin{aligned}\lambda_I = \int_{\rm cell} d^3x \Bigg\{ &\frac{f_\pi^2}{6}\left(\frac{\chi}{f_\chi}\right)^2 \left[3 - \frac{1}{2}{\rm tr}(U_0\tau_a U_0^\dagger \tau_a)\right] \\ &+ \frac{1}{24e^2}\Bigg[\left[3 - \frac{1}{2}{\rm tr}(U_0\tau_a U_0^\dagger\tau_a)\right]{\rm tr}(\partial_i U_0^\dagger \partial_i U_0) \\ &+ {\rm tr}(\partial_i U_0 \tau_a \partial_i U_0^\dagger \tau_a) \\ &+ \frac{1}{2}{\rm tr}(\partial_i U_0 U_0^\dagger \tau_a \partial_i U_0 U_0^\dagger \tau_a) \\ &+ \frac{1}{2}{\rm tr}(\partial_i U_0^\dagger U_0 \tau_a \partial_i U_0^\dagger U_0 \tau_a)\Bigg]\Bigg\}.\end{aligned}$$

$I^{\rm tot}$ is the total isospin, which would be the same as the third component of the isospin I_3 for pure neutron matter. This suggests taking for $\delta \equiv (N-P)/(N+P) \lesssim 1$

$$I^{\rm tot} = \tfrac{1}{2}A\delta. \tag{14}$$

Thus, the energy per nucleon in an infinite matter ($A=\infty$) is

$$E = E_0 + \frac{1}{8\lambda_I}\delta^2, \tag{15}$$

with $E_0 = M_{cl}$. This leads to the symmetry energy

$$E_{\rm sym} = \frac{1}{8\lambda_I}. \tag{16}$$

The numerical results for the given parameters are plotted for densities below and above n_0 in Fig. 4. The Klebanov collective quantization method is not expected to be applicable for very low densities. The refined treatment made in Ref. [29] with the Skyrme model (using the rational-map approximation) of mass splittings of nuclear isotopes shows that the $E_{\rm sym}$ in finite systems decreases as the mass number A increases. This implies that the decrease in $E_{\rm sym}$ seen just below n_0 in Fig. 4 is consistent with the result of Ref. [29]. The striking feature, that is, the cusp at $n_{1/2}$, reproduces what is expected with the new scaling relations (6) and (11) as schematically shown in Fig. 3.

One might raise an objection at this point to the reliability of calculating the symmetry energy $E_{\rm sym}$ as a collective quantization correction. With the Lagrangian (1), it is very likely that the symmetric part of the energy of the nucleonic matter, that is, E_0 in Eq. (15), will not saturate at the correct density. This is because there is not enough repulsion in the model that would balance the attraction coming from the dilaton, that is, the mechanism that reduces the soliton mass to ~1 GeV in [13], near the saturation density.

We argue that this problem does not affect our conclusion. Clearly the ω field, when implemented as in Ref. [29], will remove this defect. What we are calculating for $E_{\rm sym}$ is a $1/N_c$ effect like the $N-\Delta$ mass difference since it is $\propto 1/\lambda_I$ and the moment of inertia $\lambda_I \propto N_c$. What enters into λ_I is the leading N_c term, and the subleading effects that figure in the saturation in E_0 would not affect $E_{\rm sym}$ to the leading order we are considering. An evidence for this is the fact that the location of the density $n_{1/2}$ is extremely insensitive to the dilaton mass as one can see in Fig. 3 of Ref. [17], whereas the E_0 (hence the density n_χ at which $\langle\bar{q}\rangle^* = f_\pi^* = 0$) is strongly affected by the dilaton mass.

VIII. FURTHER COMMENTS

We should mention that the model used in this article has several caveats that need to be addressed. For instance, large N_c arguments invoked for the crystal structure of dense matter may be invalidated by $1/N_c$ (quantum) corrections. The quantum fluctuations could melt the crystal, turning the Skyrmion crystal into a Skrymion liquid. Zahed discussed what happens to the dyonic salt crystal in holographic QCD when the system is heated and arrived at a low temperature of ~10 MeV for the system to melt into a dyonic liquid [30]. Quantum fluctuation and thermal fluctuation are expected to act in a similar way. The question would be whether this melting invalidates the argument for the change of the scaling in the density regime we have considered and its impact on the tensor forces. We have no clear answer to this. However we would conjecture that since the transition from the Skyrmion matter to the half-Skyrmion matter is topological, the qualitative feature would survive owing to topological stability.

A plausible consequence of the new scaling proposed in this paper is the formation of a neutron solid with π^0 condensate discussed a long time ago [31]. In [31], Pandharipande and Smith argued that with certain enhancement of the pion tensor force, the crystal structure with π^0 condensate should be more favored energetically than liquid structure. Since our new scaling makes the ρ tensor suppressed while the pion tensor is left strong at high density, the half-Skyrmion phase in solid form could be *a posteriori* justified.

It seems possible that a continuum description of the half-Skyrmion matter suitable for liquid structure at moderate density—prior to the possible π^0 condensation—is related to Georgi's vector symmetry conjectured to be present in the large N_c limit [32]. That the chiral symmetry is putatively restored with $\mathrm{Tr}U = 0$ but the pion decay constant remains nonzero [33] may be interpreted in hidden local symmetry theory in terms of $f_\pi \neq 0$ and $a = 1$. We should point out that it also resembles the "hadronic freedom" regime invoked for the region of hot/dense matter between the "flash point"—at which hadrons in medium go ∼90% on-shell—and the chiral transition point in which the vector coupling $g_V \approx 0$, with the hadronic interactions becoming weak [34]. This issue calls for a rigorous treatment.

Also, the Skyrme model supplemented with the soft dilaton only and without the ω meson degree of freedom—which behaves as a six-derivative term in Ref. [29]—may be too simplistic. Some of those degrees of freedom that are integrated out, such as the tower of vector mesons including the ω meson, may have to be considered explicitly. The extreme sensitivity of the phase transition point n_c (in contrast to $n_{1/2}$) to the dilaton mass may be a signal for this. Nonetheless, the picture we have obtained for the tensor forces and the symmetry energy seems to be consistent and qualitatively robust. This may have to do with the topological nature of the transition involved as in certain condensed-matter processes.

The present formulation offers a possibility of determining the symmetry energy for neutron-star matter even with hyperons present. This could be done by collective-quantizing multikaons bound in the Skyrmion matter constrained with β equilibrium. This would also allow one to study dense multikaonic nuclear matter along the line discussed in Ref. [1] for one antikaon.

We stress that the forthcoming experiments at RIB and FAIR could check the anomalous behavior of the symmetry energy at $n_{1/2}$ and, if present, determine $n_{1/2}$. It is amusing to note that such measurements would also pin down the constant e and possibly other parameters if introduced in the effective chiral Lagrangian. If it turned out that $n_{1/2}$ were not too high above n_0, it should be no exaggeration to state that the scaling proposed here would have strong consequences on *all* processes that probe highly compressed cold matter, say, at FAIR.

ACKNOWLEDGMENTS

We are grateful for useful correspondence with Bao-An Li and Chang Xu on the symmetry energy, especially on their work in Ref. [24]. This work was supported by the WCU Project of Korean Ministry of Education, Science, and Technology (No. R33-2008-000-10087-0).

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