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Research Article

Radical Structures of Fuzzy Polynomial Ideals in a Ring

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We investigate the radical structure of a fuzzy polynomial ideal induced by a fuzzy ideal of a ring and study its properties. Given a fuzzy ideal β of R and a homomorphism $f : R \to R'$, we show that if f_x is the induced homomorphism of f, that is, $f_x(\sum_{i=0}^{n} a_i x^i) = \sum_{i=0}^{n} f(a_i)x^i$, then ${f_x}^{-1}[(\sqrt{\beta})_x] = (\sqrt{f^{-1}(\beta)})_x$.

1. Introduction

Zadeh [1] introduced the notion of a fuzzy subset A of a set X as a function from X into $[0, 1]$. Rosenfeld [2] applied this concept to the theory of groupoids and groups. Liu [3] introduced and studied the notion of the fuzzy ideals of a ring. Following Liu, Mukherjee and Sen [4] defined and examined the fuzzy prime ideals of a ring. The concept of fuzzy ideals was applied to several algebras: BN-algebras [5], BL-algebras [6], semirings [7], and semigroups [8]. Ersoy et al. [9] applied the concept of intuitionistic fuzzy soft sets to rings, and Shah et al. [10] discussed intuitionistic fuzzy normal subrings over a nonassociative ring. Prajapati [11] investigated residual of ideals of an L-ring. Dheena and Mohanraj [12] obtained a condition for a fuzzy small right ideal to be fuzzy small prime right ideal.

The present authors [13] introduced the notion of a fuzzy polynomial ideal α_x of a polynomial ring $R[x]$ induced by a fuzzy ideal α of a ring R and obtained an isomorphism theorem of a ring of fuzzy cosets of α_x. It was shown that a fuzzy ideal α of a ring is fuzzy prime if and only if α_x is a fuzzy prime ideal of $R[x]$. Moreover, we showed that if α_x is a fuzzy maximal ideal of $R[x]$, then α is a fuzzy maximal ideal of R.

In this paper we investigate the radical structure of a fuzzy polynomial ideal induced by a fuzzy ideal of a ring and study their properties.

2. Preliminaries

In this section, we review some definitions which will be used in the later section. Throughout this paper unless stated otherwise all rings are commutative rings with identity.

Definition 1 (see [3]). A *fuzzy ideal* of a ring R is a function $\alpha : R \to [0, 1]$ satisfying the following axioms:

(1) $\alpha(x + y) \geq \min\{\alpha(x), \alpha(y)\}$.

(2) $\alpha(xy) \geq \max\{\alpha(x), \alpha(y)\}$.

(3) $\alpha(-x) = \alpha(x)$

for any $x, y \in R$.

Definition 2 (see [2]). Let $f : R \to S$ be a homomorphism of rings and let β be a fuzzy subset of S. We define a fuzzy subset $f^{-1}\beta$ of R by $f^{-1}\beta(x) := \beta(f(x))$ for all $x \in R$.

Definition 3 (see [2]). Let $f : R \to S$ be a homomorphism of rings and let α be a fuzzy subset of R. We define a fuzzy subset $f(\alpha)$ of S by

$$f(\alpha)(y) := \begin{cases} \sup\{\alpha(t) \mid t \in R, f(t) = y\} & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{if } f^{-1}(y) = \emptyset. \end{cases} \quad (1)$$

Definition 4 (see [2]). Let R and S be any sets and let $f : R \to S$ be a function. A fuzzy subset α of R is called an *f-invariant* if $f(x) = f(y)$ implies $\alpha(x) = \alpha(y)$, where $x, y \in R$.

Zadeh [1] defined the following notions. The union of two fuzzy subsets α and β of a set S, denoted by $\alpha \cup \beta$, is a fuzzy subset of S defined by

$$(\alpha \cup \beta)(x) := \max\{\alpha(x), \beta(x)\} \tag{2}$$

for all $x \in S$.

The intersection of α and β, symbolized by $\alpha \cap \beta$, is a fuzzy subset of S, defined by

$$(\alpha \cap \beta)(x) := \min\{\alpha(x), \beta(x)\} \tag{3}$$

for all $x \in S$.

Theorem 5 (see [13]). *Let $\alpha : R \to [0,1]$ be a fuzzy ideal of a ring R and let $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ be a polynomial in $R[x]$. Define a fuzzy set $\alpha_x : R[x] \to [0,1]$ by $\alpha_x(f(x)) := \min_i\{\alpha(a_i) \mid a_i$'s are coefficients of $f(x)\}$. Then α_x is a fuzzy ideal of $R[x]$.*

The fuzzy ideal α_x discussed in Theorem 5 is called the *fuzzy polynomial ideal* [13] of $R[x]$ induced by a fuzzy ideal α.

Theorem 6 (see [13]). *Let $\alpha : R \to [0,1]$ be a fuzzy ideal of a ring R. Then α is a fuzzy prime ideal of R if and only if α_x is a fuzzy prime ideal of $R[x]$.*

Notation 1. Let $\alpha : R \to [0,1]$ be a fuzzy subset of a set R. We denote a level set α_* by $\alpha_* := \{a \in R \mid \alpha(a) = \alpha(0)\}$, and we know that $\alpha(0) \geqq \alpha(x)$ for all $x \in R$. The set of all polynomials $f(x) = \sum_{i=0}^{m} a_ix^i \in R[x]$ whose α's values $\alpha(a_i)$ are equal to $\alpha(0)$ for all $i = 0, 1, \ldots, n$, is denoted by $\alpha_*[x]$.

Theorem 7 (see [13]). *If α and β are fuzzy ideals of a ring R, then*

(i) $(\alpha \cap \beta)_x = \alpha_x \cap \beta_x$,

(ii) $\alpha_x \cup \beta_x \subseteq (\alpha \cup \beta)_x$.

Let $f : R \to R'$ be a homomorphism of rings. A map $f_x : R[x] \to R'[x]$ defined by $f_x(a_0 + a_1x + \cdots + a_nx^n) := f(a_0) + f(a_1)x + \cdots + f(a_n)x^n$ is obviously a ring homomorphism, and we call it an *induced homomorphism* [13] by f.

Theorem 8 (see [13]). *Let $f : R \to R'$ be an epimorphism of rings and let f_x be an induced homomorphism of f. If α is an f-invariant fuzzy ideal of R, then $(f(\alpha))_x = f_x(\alpha_x)$.*

3. Fuzzy Polynomial Ideals

In this section, we study some relations between the radical of the fuzzy polynomial induced by a fuzzy ideal and the radical of a fuzzy ideal of a ring.

A fuzzy ideal $\alpha : R \to [0,1]$ of a ring R is called a *fuzzy prime ideal* [14] of R if α_* is a prime ideal of R. A fuzzy set $\sqrt{\alpha} : R \to [0,1]$, defined as $\sqrt{\alpha}(a) := \bigvee\{\alpha(a^n) \mid n > 0\}$, is called a *fuzzy nil radical* [15] of α.

Theorem 9 (see [15]). *If $\alpha : R \to [0,1]$ is a fuzzy ideal of R, then the fuzzy set $\sqrt{\alpha}$ is a fuzzy ideal of R.*

Lemma 10 (see [15]). *If α and β are fuzzy ideals of R, then $\sqrt{\alpha \cap \beta} = \sqrt{\alpha} \cap \sqrt{\beta}$.*

Lemma 11. *If α and β are fuzzy ideals of R, then $\sqrt{\alpha \cup \beta} = \sqrt{\alpha} \cup \sqrt{\beta}$.*

Proof. If a is an element of R, then

$$\begin{aligned}
\sqrt{\alpha \cup \beta}(a) &= \bigvee\{(\alpha \cup \beta)(a^n) \mid n > 0\} \\
&= \bigvee\{\max\{\alpha(a^n), \beta(a^n)\} \mid n > 0\} \\
&= \max\left\{\bigvee\{\alpha(a^n) \mid n > 0\}, \bigvee\{\beta(a^n) \mid n > 0\}\right\} \\
&= \max\left\{\sqrt{\alpha}(a), \sqrt{\beta}(a)\right\} = \left(\sqrt{\alpha} \cup \sqrt{\beta}\right)(a).
\end{aligned} \tag{4}$$

This proves that $\sqrt{\alpha \cup \beta} = \sqrt{\alpha} \cup \sqrt{\beta}$. □

Since α_x is a fuzzy ideal of a polynomial ring $R[x]$ by Theorem 5, the fuzzy set $\sqrt{\alpha_x} : R[x] \to [0,1]$ is the fuzzy nil radical of α_x. The following theorem gives that the two fuzzy nil radicals have the same value.

Theorem 12. *If $\alpha : R \to [0,1]$ is a fuzzy ideal of R, then*

$$(\sqrt{\alpha_x})_x = (\sqrt{\alpha})_x. \tag{5}$$

Proof. Let $f(x) := \sum_{i=0}^{m} \in R[x]$ be any element of $R[x]$. Then, by Theorem 5, we have $\alpha_x(a_j^n) = \alpha_x(a_j^n + 0x + 0x^2 + \cdots + 0x^m) = \min\{\alpha(a_j^n), \alpha(0), \ldots, \alpha(0)\} = \alpha(a_j^n)$. Since $\sqrt{\alpha_x}$ is a fuzzy ideal of $R[x]$, we obtain

$$\begin{aligned}
(\sqrt{\alpha_x})_x(f(x)) &= \min_{i=0}^{m}\{\sqrt{\alpha_x}(a_i)\} \\
&= \min_{i=0}^{m}\left\{\bigvee\{\alpha_x(a_i^n) \mid n > 0\}\right\} \\
&= \min_{i=0}^{m}\left\{\bigvee\{\alpha(a_i^n) \mid n > 0\}\right\} \\
&= \min_{i=0}^{m}\{\sqrt{\alpha}(a_i)\} = (\sqrt{\alpha})_x(f(x)).
\end{aligned} \tag{6}$$

This proves that $(\sqrt{\alpha})_x = (\sqrt{\alpha_x})_x$. □

Theorem 13. *If α and β are fuzzy ideals of R, then*

$$\left(\sqrt{\alpha \cap \beta}\right)_x = (\sqrt{\alpha})_x \cap \left(\sqrt{\beta}\right)_x. \tag{7}$$

Proof. If α and β are fuzzy ideals of R, then α_x and β_x are fuzzy ideals of $R[x]$ by Theorem 5. It follows from Theorems 12 and 7(i) and Lemma 10 that

$$\begin{aligned}\left(\sqrt{\alpha\cap\beta}\right)_x &= \left(\sqrt{(\alpha\cap\beta)_x}\right)_x \quad \text{[Theorem 12]}\\ &= \left(\sqrt{\alpha_x\cap\beta_x}\right)_x \quad \text{[Theorem 7 (i)]}\\ &= \left(\sqrt{\alpha_x}\cap\sqrt{\beta_x}\right)_x \quad \text{[Lemma 10]}\\ &= (\sqrt{\alpha_x})_x\cap\left(\sqrt{\beta_x}\right)_x \quad \text{[Theorem 7 (i)]}\\ &= (\sqrt{\alpha})_x\cap\left(\sqrt{\beta}\right)_x \quad \text{[Theorem 12]}\end{aligned} \tag{8}$$

proving the theorem. □

Theorem 14. *If α and β are fuzzy ideals of R, then*

$$(\sqrt{\alpha})_x\cup\left(\sqrt{\beta}\right)_x\subseteq\left(\sqrt{\alpha\cup\beta}\right)_x. \tag{9}$$

Proof. Since α_x and β_x are fuzzy ideals of $R[x]$ by Theorem 5, we obtain

$$\begin{aligned}(\sqrt{\alpha})_x\cup\left(\sqrt{\beta}\right)_x &= (\sqrt{\alpha_x})_x\cup\left(\sqrt{\beta_x}\right)_x \quad \text{[Theorem 12]}\\ &\subseteq \left(\sqrt{\alpha_x}\cup\sqrt{\beta_x}\right)_x \quad \text{[Theorem 7 (ii)]}\\ &= \left(\sqrt{\alpha_x\cup\beta_x}\right)_x \quad \text{[Lemma 11]}\\ &\subseteq \left(\sqrt{(\alpha\cup\beta)_x}\right)_x \quad \text{[Theorem 7 (ii)]}\\ &= \left(\sqrt{\alpha\cup\beta}\right)_x \quad \text{[Theorem 12]}.\end{aligned} \tag{10}$$

This proves that $(\sqrt{\alpha})_x\cup(\sqrt{\beta})_x\subseteq(\sqrt{\alpha\cup\beta})_x$. □

Theorem 15. *Let β be a fuzzy ideal of R and let $f: R\to R'$ be a homomorphism of rings. If f_x is the induced homomorphism of f, that is, $f_x(\sum_{i=0}^n a_ix^i)=\sum_{i=0}^n f(a_i)x^i$, then*

$$f_x^{-1}\left[\left(\sqrt{\beta}\right)_x\right]=\left(\sqrt{f^{-1}(\beta)}\right)_x. \tag{11}$$

Proof. Given a polynomial $g(x)=b_0+b_1x+\cdots+b_mx^m\in R[x]$, we have

$$\begin{aligned}\left(\sqrt{f^{-1}(\beta)}\right)_x(g(x)) &= \min\left\{\sqrt{f^{-1}\beta}(b_0),\right.\\ &\left.\sqrt{f^{-1}\beta}(b_1),\ldots,\sqrt{f^{-1}\beta}(b_m)\right\}\\ &= \min\left\{\bigvee\left\{f^{-1}\beta(b_0{}^n)\mid n>0\right\},\right.\\ &\bigvee\left\{f^{-1}\beta(b_1{}^n)\mid n>0\right\},\ldots,\\ &\left.\bigvee\left\{f^{-1}\beta(b_m{}^n)\mid n>0\right\}\right\}\\ &= \min\left\{\bigvee\left\{\beta\left(f(b_0)^n\right)\mid n>0\right\},\right.\\ &\bigvee\left\{\beta\left(f(b_1)^n\right)\mid n>0\right\},\ldots,\\ &\left.\bigvee\left\{\beta\left(f(b_m)^n\right)\mid n>0\right\}\right\}=\min\left\{\sqrt{\beta}(f(b_0)),\right.\\ &\left.\sqrt{\beta}(f(b_1)),\ldots,\sqrt{\beta}(f(b_m))\right\}=\left(\sqrt{\beta}\right)_x\\ &\cdot(f_x(g(x)))=f_x^{-1}\left[\left(\sqrt{\beta}\right)_x\right](g(x)).\end{aligned} \tag{12}$$

This proves that $f_x^{-1}[(\sqrt{\beta})_x]=(\sqrt{f^{-1}(\beta)})_x$. □

Proposition 16 (see [15]). *Let $f: R\to R'$ be a ring epimorphism from R onto R', and let $\alpha: R\to[0,1]$ be a fuzzy ideal of R. If α is constant on* Ker f, *then $f(\sqrt{\alpha})=\sqrt{f(\alpha)}$.*

Theorem 17. *Let $f: R\to R'$ be a homomorphism of rings and let f_x be the induced homomorphism of f. If a fuzzy ideal α of R is constant on* Ker f, *then the fuzzy polynomial ideal α_x is constant on* Ker f_x.

Proof. Let $\alpha(a)=k_0$ for all $a\in$ Ker f and let $g(x)=b_0+b_1x+\cdots+b_mx^m$ be any element of Ker f_x. Then $0=f_x(g(x))=f(b_0)+f(b_1)x+\cdots+f(b_m)x^m$. It follows that $f(b_i)=0$ for all $i=0,1,\ldots,m$. Hence $b_i\in$ Ker f for all $i=0,1,\ldots,m$; that is, $\alpha(b_i)=k_0$ for all $i=0,1,\ldots,m$. This shows that $\alpha_x(g(x))=\min\{\alpha(b_0),\alpha(b_1),\ldots,\alpha(b_m)\}=k_0$, proving the theorem. □

Corollary 18. *Let $f: R\to R'$ be an epimorphism of rings and let f_x be the induced homomorphism of f. If an f-invariant fuzzy ideal α of R is constant on* Ker f, *then*

$$f_x(\sqrt{\alpha_x})=\sqrt{(f(\alpha))_x}. \tag{13}$$

Proof. It follows from Proposition 16 and Theorem 8 that $f_x(\sqrt{\alpha_x})=\sqrt{f_x(\alpha_x)}=\sqrt{(f(\alpha))_x}$. □

Theorem 19. *Let α be a fuzzy ideal of R and let α_x be its fuzzy polynomial ideal of $R[x]$. If β is a fuzzy prime ideal of $R[x]$ such that $\alpha_x\subsetneqq\beta$, then there exists a fuzzy prime ideal α_0 of R such that $(\alpha_0)_*=\beta_*\cap R$ and $\alpha\subsetneqq\alpha_0$.*

Proof. Since β is a fuzzy prime ideal of $R[x]$, β_* is a prime ideal of $R[x]$. If we define $\gamma:=\beta_*\cap R$, then it is easy to show

that γ is a prime ideal of R. Define a fuzzy subset $\alpha_0 : R \to [0,1]$ by

$$\alpha_0(a) = \begin{cases} \beta(0) & \text{if } a \in \gamma, \\ \beta(a) & \text{if } a \notin \gamma. \end{cases} \tag{14}$$

Then, by routine calculations, we show that α_0 is a fuzzy ideal of R satisfying $(\alpha_0)_* = \gamma$. We claim that $\alpha \subseteqq \alpha_0$. Given $a \in R$, if $a \in \gamma$, then $\alpha(a) = \alpha_x(a) \le \beta(a) \le \beta(0) = \alpha_0(a)$. If $a \notin \gamma$, then $\alpha(a) = \alpha_x(a) \le \beta(a) = \alpha_0(a)$. Since γ is a prime ideal of R, $(\alpha_0)_*$ is a prime ideal of R. This shows that α_0 is a fuzzy prime ideal of R, proving the theorem. □

Definition 20 (see [15]). Let $\alpha : R \to [0,1]$ be a fuzzy ideal of R. The fuzzy ideal $r(\alpha)$ defined by

$$r(\alpha) := \bigcap \{\beta \mid \alpha \subseteqq \beta,\ \beta : \text{a fuzzy prime ideal of } R\} \tag{15}$$

is called the *prime fuzzy radical* of α.

Theorem 21. *Let α be a fuzzy ideal of R and let α_x be its fuzzy polynomial ideal of $R[x]$. Then*

$$r(\alpha_x) \subseteq (r(\alpha))_x. \tag{16}$$

Proof. By Theorem 6, β_i is a fuzzy prime ideal of R with $\alpha \subseteq \beta_i$ if and only if $(\beta_i)_x$ is a fuzzy prime ideal of $R[x]$ with $\alpha_x \subseteq (\beta_i)_x$. It follows from Theorem 7(i) that

$$\begin{aligned} (r(\alpha))_x &= \Big(\bigcap \{\beta_i \mid \alpha \\ &\quad \subseteq \beta_i,\ \beta_i \text{ is a fuzzy prime ideal of } R\}\Big)_x \\ &= \Big(\bigcap \{(\beta_i)_x \mid \alpha \\ &\quad \subseteq \beta_i,\ \beta_i \text{ is a fuzzy prime ideal of } R\}\Big) = \bigcap \{(\beta_i)_x \mid \\ &\quad \alpha_x \subseteq (\beta_i)_x,\ (\beta_i)_x \text{ is a fuzzy prime ideal of } R[x]\} \\ &\quad \supseteq \bigcap \{\gamma_i \mid \alpha_x \subseteq \gamma_i, \\ &\quad \gamma_i \text{ is a fuzzy prime ideal of } R[x]\} = r(\alpha_x), \end{aligned} \tag{17}$$

proving the theorem. □

Notation 2. Let α be a fuzzy ideal of R and let α_x be its fuzzy polynomial ideal of $R[x]$. We denote $\mathrm{FPI}(\alpha)$ by

$$\mathrm{FPI}(\alpha) := \{\beta \mid \alpha \subseteq \beta,\ \beta \text{ is a fuzzy prime ideal of } R\} \tag{18}$$

and $\mathrm{FPI}(\alpha_x)$ by

$$\begin{aligned} &\mathrm{FPI}(\alpha_x) \\ &\quad := \{\gamma \mid \alpha_x \subseteq \gamma,\ \gamma \text{ is a fuzzy prime ideal of } R[x]\}. \end{aligned} \tag{19}$$

Theorem 22. *Let α be a fuzzy ideal of R and let α_x be its fuzzy polynomial ideal of $R[x]$. Then a map $\phi : \mathrm{FPI}(\alpha) \to \mathrm{FPI}(\alpha_x)$ defined by $\phi(\beta) := \beta_x$ is one-one.*

Proof. If $\beta, \gamma \in \mathrm{FPI}(\alpha)$ such that $\phi(\beta) = \phi(\gamma)$, then $\beta_x = \gamma_x$. It follows that $\beta_x(a) = \gamma_x(a)$ for all $a \in R$, and hence $\beta(a) = \gamma(a)$ for all $a \in R$, proving that $\beta = \gamma$. Hence ϕ is one-one. □

Corollary 23. *Let α be a fuzzy ideal of R and let α_x be its fuzzy polynomial ideal of $R[x]$. If the map ϕ defined in Theorem 22 is an onto map, then*

$$(r(\alpha))_x = r(\alpha_x). \tag{20}$$

Proof. If β is any element of $\mathrm{FPI}(\alpha_x)$, then there exists $\gamma \in \mathrm{FPI}(\alpha)$ such that $\gamma_x = \phi(\gamma) = \beta$ with $\alpha \subseteq \gamma$. Thus $(r(\alpha))_x = r(\alpha_x)$. This shows that the reverse inclusion in Theorem 21 holds. □

Example 24. Let $\mathbf{Z}$ be set of all integers. Let

$$\alpha(x) := \begin{cases} 1 & \text{if } x \in 2\mathbf{Z}, \\ 0 & \text{if } x \notin 2\mathbf{Z}. \end{cases} \tag{21}$$

Then α is a fuzzy prime ideal of $\mathbf{Z}$, since $\alpha_* = 2\mathbf{Z}$ is a prime ideal of $\mathbf{Z}$, and its induced polynomial ideal α_x is

$$\alpha_x(f(x)) := \begin{cases} 1 & \text{if } f(x) \in 2\mathbf{Z}[x], \\ 0 & \text{if } f(x) \notin 2\mathbf{Z}[x]. \end{cases} \tag{22}$$

By Theorem 6, the fuzzy polynomial ideal α_x induced by α is a fuzzy prime ideal of $\mathbf{Z}[x]$. Hence $(r(\alpha))_x = \alpha_x = r(\alpha_x)$.

Competing Interests

The authors declare that they have no competing interests.

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