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Effects of Yield and Lead-Time Uncertainty on Retailer-Managed and Vendor-Managed Inventory Management

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ABSTRACT Generally, there are various elements of uncertainty in a supply chain. In particular, uncertainties in lead time, demand, and yield are very important in the semiconductor industry. Higher uncertainty can lead to bullwhip effects that can undermine the performance of the entire supply chain. This study examines the relationship between uncertainty in the supply chain and the outcome of inventory replenishment policies. Specifically, we analyze the effects of well-known uncertainties on manufacturer production quantity and retailer order quantity decisions in a decentralized supply chain. In addition, we also analyze and compare the effects of these uncertainties for the retailer-managed inventory and the vendor-managed inventory policies. Using numerical experiments, a comparative analysis of the two alternatives is conducted to determine suitable options for improving supply chain performance. In general, the performance of vendor-managed inventory is better than that of retailer-managed inventory, but we observe from the numerical experiments that there exist circumstances under which retailer-managed inventory shows better supply chain performance.

INDEX TERMS Yield, lead-time, vendor-managed inventory, retailer-managed inventory, decentralized supply chain, optimal production quantity, optimal order quantity, single-period inventory.

I. INTRODUCTION

Semiconductors are the core components of a variety of electronic devices, such as smartphones and tablet PCs. Due to the recent high demand for mobile devices, the demand for semiconductors has also risen. In addition, as more firms require electronic devices to store and collect big data for analysis purposes, the demand for semiconductors has reached record highs over the past few years. Although the long-term demand for semiconductors has been increasing, short-term demand remains difficult for semiconductor manufacturers to predict. As Brown *et al.* [1] noted, semiconductors are usually used as components of other products. Therefore, because semiconductor manufacturers remain at the top of the supply chain, manufactured semiconductors enter several supply chains before they finally reach consumers. During this process, demand information is frequently distorted, which eventually leads to unexpected bullwhip effects in a supply chain [2]. In researching the relationship between demand uncertainty and the bullwhip effect, Zotteri [3] examined the impact of the bullwhip effect by comparing the sell-in and sell-out quantities of a wide range of personal care products using simulations. The results showed that the bullwhip effect caused by uncertainty in short-term demand led to significant differences between the sell-in and sell-out quantities of up to 390%. Many studies have thus been conducted to minimize the bullwhip effects caused by demand uncertainty. Some have attempted to estimate demand with higher accuracy [4], [5], while others have investigated information sharing that allows producers to quickly receive demand information [6]–[8].

Semiconductor production is highly dependent on the yield of the manufacturing process. According to Shin and Park [9], the yield of the production process

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is determined by numerous parameters. Accounting for more than 23.5% of the semiconductor memory market, South Korea is the top semiconductor memory manufacturer in the world [10]. However, recent trade restrictions introduced by Japan on important materials such as hydrogen fluoride and fluorinated polyimides to South Korea have posed serious threat to its semiconductor supply chain [11]. Hydrogen fluoride is one of the most important materials used in the etching process for semiconductor manufacturing [12]. In order to maximize yield, firms have typically imported hydrogen fluoride with a purity of 99.999% or higher from Japan [13].

Taking all of these factors into consideration, number of models have been proposed to overcome yield issues. Gardner *et al.* [14] discovered an improvement in production cycle times when yields improved. Later, Radojcic and Rencher [15] noted that yield improvements can increase the likelihood of on-time product delivery. Due to the importance of yield, Kumar *et al.* [16] attempted to construct a model for predicting semiconductor yields. However, even though the proposed model analyzed potential yield using a variety of probability distributions, it failed to account for lead time and demand. In addition, numerous papers have simultaneously considered both yield uncertainty and demand uncertainty to determine optimal inventory decisions [17]–[20].

TABLE 1. Global wafer fabrication capacity [21].

Country/Region	2015	2016	2018
Taiwan	21.7%	21.3%	21.8%
South Korea	20.5%	20.9%	21.3%
Japan	17.3%	17.1%	16.8%
North America	14.2%	13.4%	12.7%
China	9.7%	10.8%	6.0%
Europe	6.4%	6.4%	6.0%
Rest of World	10.2%	10.1%	8.7%

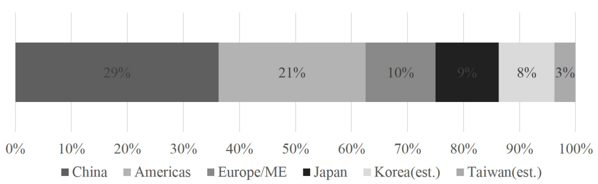


FIGURE 1. Key markets for semiconductors 2015, Sales, $ Billions [25].

Another factor that complicates the supply chain as global supply chains proliferate is lead time uncertainty. As Table 1 shows, approximately 76% of all semiconductor manufacturing facilities are centered in Asia. Figure 1 also shows that the U.S. and the EU which account for one-third of the semiconductor market, are located far away from the production facilities in Asia. In other words, most semiconductor products are produced in Asia but consumed outside of Asia. This long distance can lead to very long delivery times, which contribute to an increase in lead-time uncertainty. Lee *et al.* [22] investigated several factors, including lead-time uncertainty and pollution costs from transportation vehicles, to determine the optimal order quantity. The results showed that, due to the increase in uncertainty with longer delivery distances, it would occasionally be more cost-effective to source goods domestically rather than internationally. Song *et al.* [23] analyzed lead-time uncertainty using lead-time duration and variability. They showed that, as lead-time uncertainty decreases, the reorder point and order quantity also decrease. Hence, lead-time uncertainty contributes to higher inventory costs by increasing the average inventory level. Acar *et al.* [24] analyzed lead-time uncertainty using simulation models. By using statistical testing, this research confirmed that, as lead-time uncertainty increases, inventory costs also increase. In fact, the results showed that inventory costs increased by 5.1% due to lead-time uncertainty.

This uncertainty is further increased when national issues such as trade disputes occur. The stricter export screening process between South Korea and Japan will not only cause problems in supplying raw materials but will also lead to frequent delays in the supply chain, which then eventually increase lead-time uncertainty [11]. There have been several studies evaluating the impact of lead-time uncertainty in a supply chain. For example, Hnaien *et al.* [26] developed a one-period inventory model for a one-level assembly system based on stochastic demand and lead times. He determined the optimal quantity and optimal planned lead times using a mathematical model based on branch and bound. Chaharsooghi and Heydari [27] also developed a multi-level linear supply chain model. He discovered that the variance in lead-time has bigger impact than the mean lead time. He used a simulation to model different lead time variances to examine the performance of the system.

In the face of greater uncertainty, retailers must order more than necessary in order to avoid possible shortages. Manufactures might then misinterpret this phenomenon as an increase in demand and increase production to meet a nonexistent increase in demand. This situation is commonly known as the bullwhip effect, and is the source of many problems in the supply chain that eventually decrease overall efficiency. In order to ameliorate the bullwhip effect, many markets, under the assumption of a decentralized supply chain, have adopted the vendor managed inventory (VMI) model in place of the retailer managed inventory (RMI) model. The VMI model differs from the RMI model in that the vendor, not the retailer, manages the retailer's inventory. Disney and Towill [28] investigated the bullwhip effect for a single retailer and a single manufacture in a supply chain using a simulation model. It was found that VMI was better at responding to demand uncertainty due to discount ordering or price changes. However, the focus was on the effect of demand variability, hence there was no consideration of lead time or yield. Fry *et al.* [29] analyzed a (Z, z)-type VMI contract between a single supplier and a single retailer in a supply chain, focusing on the behavior of the supplier and retailer under the VMI and RMI models. They reported that the penalties for understocking for the supplier are not incurred immediately, but they do effect long-term performance.

It is well-known that the VMI model is generally preferred to the RMI model for the supply chain as a whole, but no research has simultaneously investigated the benefits of the VMI model for various forms of uncertainty (yield, lead-time, demand, etc).

This paper focuses on the demand, yield, and lead-time uncertainty in the supply chain. To the best of our knowledge, studies that simultaneously deal with demand, lead time, and yield uncertainty are scarce, thus we believe that this paper will be more applicable to a wide range of circumstances.

The rest of the paper is organized as follows. Section II presents the centralized supply chain model as a benchmark. Sections III and IV compare the optimal order and production quantity for the RMI and VMI models, respectively in a decentralized supply chain. In Section V, we perform a set of numerical experiments that highlight certain factors that affect the behavior of retailers and manufacturers in a variety of operating environments. Finally, Section VI summarizes our findings and concludes our discussion.

II. BENCHMARK: CENTRALIZED SUPPLY CHAIN

In this section, we investigate the centralized (or vertically integrated) supply chain model as a benchmark for better understanding the impacts of the RMI and VMI strategies in decentralized supply chains. In this centralized setting, the centralized firm determines initial production quantity Z in the presence of supply (particularly, yield and lead-time) uncertainty before the season begins. We let ξ be a random variable with c.d.f $G(\cdot)$ with support of $[\underline{\xi}, \overline{\xi}]$, representing the random yield rate, and adopt the multiplicative random yield model, which frequently appears in the literature. In addition, we let L be a random variable with c.d.f $H(\cdot)$ representing the lead-time uncertainty. We note that $L < 0$ ($L > 0$) indicates that the production completes before (after) the selling season, and $|L|$ represents the time since the beginning of the selling season. Other notations are given in Table 2.

TABLE 2. Notation.

c	unit production cost
p	unit retail price; $p > c$
h	unit holding cost per unit period
s	unit salvage value; $s < c$
X	random demand with c.d.f. $F(\cdot)$
$\xi \in [\underline{\xi}, \overline{\xi}]$	random yield rate with c.d.f. $G(\cdot)$
L	random lead-time with c.d.f. $H(\cdot)$
Z	production quantity (decision variable)
$(a)^+$	$= \max\{a, 0\}$

We now present the expected profit function $\Pi_C(Z)$ given production quantity Z for the centralized firm, shown in Equation (1) at the bottom of this page.

As expressed in Equation (1), the unit production cost c is incurred for the realized yield, which can be sold at retail price p. If the demand X is less than the sellable production quantity ξZ, then the firm salvages $\xi Z - X$ unsold products at unit salvage value s. We further assume that, if the manufacturing department completes the production of the order earlier than the start date of the selling season, the firm suffers the extra costs of inventory holding until the beginning of the selling season with unit holding cost h per unit time. We note that the negative sign immediately before h in ($\star$) from Equation (1) is due to the negativeness of l. On the other hand, a delay in production does not incur any further penalties but instead results in the reduced sales. Let Z_C^* be the optimal production quantity that maximizes $\Pi_C(Z)$.

Proposition 1: (i) *$\Pi_C(Z)$ is concave in Z.*
(ii) *Z_C^* satisfies the following equation:*

$$\begin{aligned} &\int_{-\infty}^{0} \left(\int_{\underline{\xi}}^{\overline{\xi}} \xi F(\xi Z_C^* | l \le 0) dG(\xi) \right) dH(l) \\ &\quad + \int_{0}^{\infty} \left(\int_{\underline{\xi}}^{\overline{\xi}} \xi F(\xi Z_C^* | l \ge 0) dG(\xi) \right) dH(l) \\ &\quad = \left[\frac{(p-c) + h \int_{-\infty}^{0} l dH(l)}{p-s} \right] \mu_\xi. \end{aligned} \tag{2}$$

Proof: See Appendix A-A. □

From the result in Proposition 1, it is straightforward to show that the optimal production quantity Z_C^* for the centralized firm increases in p, s and h while it decreases in c. We remark that the term $h \int_{-\infty}^{0} l dH(l)$ is non-positive and its absolute value expresses the expected unit holding cost over the earliness of production; therefore, Z_C^* indeed decreases as the holding cost increases due to the earliness.

III. DECENTRALIZED SUPPLY CHAIN UNDER A RETAILER-MANAGED INVENTORY

We now turn our attention to decentralized supply chain models where both independent parties – the manufacturer and the retailer – pursue their own interests. Under a decentralized setting, we consider the retailer-managed inventory strategy and assume that the manufacturer follows a wholesale price scheme that depends on the lateness of order delivery as

$$\begin{aligned} \Pi_C(Z) = &\int_{-\infty}^{0} \left[\int_{\underline{\xi}}^{\overline{\xi}} \left(\int_{0}^{\xi Z} \Big(px + s(\xi Z - x)\Big) dF(x|l \le 0) + \int_{\xi Z}^{\infty} \Big(p\xi Z\Big) dF(x|l \le 0) \right) dG(\xi) - \underbrace{(c - hl)}_{(\star)} \mu_\xi Z \right] dH(l) \\ &+ \int_{0}^{\infty} \left[\int_{\underline{\xi}}^{\overline{\xi}} \left(\int_{0}^{\xi Z} \Big(px + s(\xi Z - x)\Big) dF(x|l \ge 0) + \int_{\xi Z}^{\infty} \Big(p\xi Z\Big) dF(x|l \ge 0) \right) dG(\xi) - c\mu_\xi Z \right] dH(l). \end{aligned} \tag{1}$$

follows:

$$w(l) = \begin{cases} \underline{w} & \text{if } l \le 0 \\ \underline{w} - \tau(l) & \text{if } l \ge 0 \end{cases}$$

where (i) $w(l)$ is assumed to be exogenous, which is typically acceptable in a highly competitive market, and (ii) $\tau(l)$ is nondecreasing in $l \ge 0$, which compensates for the loss due to late delivery to the retailer and is proportional to its lateness. We further assume that the expected unit wholesale price is greater than or equal to the unit salvage value (i.e., $\underline{w} - \int_0^\infty \tau(l)dH(l) \ge s$), which is a weak assumption. We discuss the impact of a change in $w(l)$ on supply chain performance based on the numerical sensitivity analysis in Section V.

In this section, we first investigate the case where the retailer takes full responsibility of its inventory management (i.e., RMI). The sequence of events in this scenario is as follows:

1) Prior to the selling season, the manufacturer takes the market wholesale price $w(l)$ in the market for the retailer.
2) The retailer determines the order quantity Q, and places an order with the manufacturer.
3) The manufacturer decides production quantity Z. Due to the uncertain yield, the sellable quantity ξZ is less than or equal to Z. Only $\min\{\xi Z, Q\}$ is delivered to the retailer, and the manufacturer salvages $(\xi Z - Q)^+$ at unit salvage value s. Due to the uncertain production completion time, if production is completed before the promised delivery date the manufacturer incurs, a holding cost per unit time, (h).
4) Customer demand X occurs in the retail channel and, at the end of the selling season, the retailer salvages $(\min\{\xi Z, Q\} - X)^+$ at unit salvage value s.

Based on the sequence of the events above, we first examine the decision problem from the manufacturer's perspective. It has to determine production quantity Z that maximizes its own expected profit given the order quantity Q from the retailer (thus, the optimal production quantity Z is a function of the order quantity Q). The expected profit function of the manufacturer, $\Pi^M_{DR}(Z;Q)$, is then expressed as Equation (3), shown at the bottom of this page.

$\Pi^M_{DR}(Z;Q)$ basically consists of two cases depending on the production completion time (i.e., one when $L \in (-\infty, 0]$ and the other when $L \in (0, +\infty)$), and each case is subsequently divided into subcases depending on the production yield (i.e., one when $\xi Z \le Q$, and the other when $\xi Z \ge Q$). The following proposition presents the characteristics of the expected profit function $\Pi^M_{DR}(Z;Q)$ and the optimal production quantity Z for the manufacturer.

Proposition 2: $\Pi^M_{DR}(Z;Q)$ is concave in Z, and hence $Z^*_{DR}(Q)$ – the best response (production quantity) for the manufacturer when the order quantity from the retailer is Q satisfies Equation (4), shown at the bottom of this page.

Proof: See Appendix A-B. □

We now investigate how the best response $Z^*_{DR}(Q)$ for the manufacturer behaves as order quantity Q from the retailer increases. For this purpose, we revisit the first-order optimality condition (Equation (4)), and evaluate the first-order derivative of Z^*_{DR} with respect to Q, which results in

$$\frac{\left(Z^*_{DR}(Q) - Q\frac{dZ^*_{DR}(Q)}{dQ}\right)Q}{Z^*_{DR}(Q)^3} \times \left(\underline{w} - \int_0^\infty \tau(l)dH(l) - s\right) g\left(\frac{Q}{Z^*_{DR}(Q)}\right) = 0. \quad (5)$$

From the equation above, $\frac{dZ^*_{DR}(Q)}{dQ} = \frac{Z^*_{DR}(Q)}{Q}$ can be obtained. By taking the second-order derivative of Z^*_{DR} with respect to Q, $\frac{d^2Z^*_{DR}(Q)}{dQ^2} = \frac{\frac{dZ^*_{DR}(Q)}{dQ}Q - Z^*_{DR}(Q)}{Q^2} = 0$ due to the earlier result on $\frac{dZ^*_{DR}(Q)}{dQ}$, implying that $\frac{dZ^*_{DR}(Q)}{dQ}$ is constant. We let ρ be such the constant that $\frac{dZ^*_{DR}(Q)}{dQ} = \rho$ or $Z^*_{DR}(Q) = \rho Q$. Then, the first-order optimality condition (Equation (4)) can be rewritten as follows:

$$(\underline{w} - c)\int_{\underline{\xi}}^{1/\rho} \xi dG(\xi) - (c - s)\int_{1/\rho}^{\overline{\xi}} \xi dG(\xi) + h\mu_\xi \int_{-\infty}^0 l dH(l) - \int_0^\infty \tau(l)\left[\int_{\underline{\xi}}^{1/\rho} \xi dG(\xi)\right] dH(l) = 0. \quad (6)$$

We now discuss the decision model for the retailer. Given the the manufacturer's best response $Z^*_{DR}(Q) = \rho Q$,

$$\Pi^M_{DR}(Z;Q) = \int_{-\infty}^0 \left[\int_{\underline{\xi}}^{Q/Z} \left(\underline{w}\xi Z\right) dG(\xi) + \int_{Q/Z}^{\overline{\xi}} \left(\underline{w}Q + s(\xi Z - Q)\right) dG(\xi) - (c - hl)\,\mu_\xi Z\right] dH(l)$$
$$+ \int_0^\infty \left[\int_{\underline{\xi}}^{Q/Z} \left((\underline{w} - \tau(l))\xi Z\right) dG(\xi) + \int_{Q/Z}^{\overline{\xi}} \left((\underline{w} - \tau(l))Q + s(\xi Z - Q)\right) dG(\xi) - c\mu_\xi Z\right] dH(l). \quad (3)$$

$$(\underline{w} - c)\int_{\underline{\xi}}^{Q/Z^*_{DR}(Q)} \xi dG(\xi) - (c - s)\int_{Q/Z^*_{DR}(Q)}^{\overline{\xi}} \xi dG(\xi) + h\mu_\xi \int_{-\infty}^0 l dH(l) - \int_0^\infty \tau(l)\left[\int_{\underline{\xi}}^{Q/Z^*_{DR}(Q)} \xi dG(\xi)\right] dH(l) = 0. \quad (4)$$

the retailer determines the order quantity that maximizes the expected profit function $\Pi^R_{DR}(Q)$, shown in Equation (7) at the bottom of this page.

Proposition 3: Given the best response $Z^*_{DR}(Q)$ for the manufacturer, the retailer's expected profit function $\Pi^R_{DR}(Q)$ is concave in Q, and the optimal order quantity Q^*_{DR} satisfies Equation (8), shown at the bottom of this page.

Proof: See Appendix A-C. □

In order to compare the centralized supply chain model and this decentralized model, we consider the total expected profit function $\Pi_{DR}(Q^*_{DR})$, given the best response Q^*_{DR} for the retailer, for the decentralized supply chain where $\Pi_{DR}(Q^*_{DR}) = \Pi^M_{DR}(Z^*_{DR}(Q^*_{DR}); Q^*_{DR}) + \Pi^R_{DR}(Q^*_{DR})$. Then,

$$\Pi_{DR}(Q^*_{DR}) = \int_{-\infty}^{0} \Pi_{DR}(Q^*_{DR}|l \le 0)dH(l) + \int_0^{\infty} \Pi_{DR}(Q^*_{DR}|l \ge 0)dH(l) \quad (9)$$

where Equations (10) and (11), shown at the bottom of this page.

The following proposition presents that the total expected profit in the centralized supply chain is higher than that in the decentralized supply chain with a retailer-managed inventory.

$$\begin{aligned}
&\Pi^R_{DR}(Q)\\
&= \int_{-\varphi b}^{0}\Bigg\{\int_{\underline{\xi}}^{1/\rho}\Big[\int_0^{\xi\rho Q}\Big(px + s(\xi\rho Q - x)\Big)dF(x|l \le 0) + \int_{\xi\rho Q}^{\infty}\Big(p\xi\rho Q\Big)dF(x|l \le 0) - \underline{w}\xi\rho Q\Big]dG(\xi)\\
&\quad + \int_{1/\rho}^{\bar{\xi}}\Big[\int_0^{Q}\Big(px + s(Q - x)\Big)dF(x|l \le 0) + \int_{Q}^{\infty}\Big(pQ\Big)dF(x|l \le 0) - \underline{w}Q\Big]dG(\xi)\Bigg\}dH(l)\\
&\quad + \int_0^{\infty}\Bigg\{\int_{\underline{\xi}}^{1/\rho}\Big[\int_0^{\xi\rho Q}\Big(px + s(\xi\rho Q - x)\Big)dF(x|l \ge 0) + \int_{\xi\rho Q}^{\infty}\Big(p\xi\rho Q\Big)dF(x|l \ge 0) - (\underline{w} - \tau(l))\xi\rho Q\Big]dG(\xi)\\
&\quad + \int_{1/\rho}^{\bar{\xi}}\Big[\int_0^{Q}\Big(px + s(Q - x)\Big)dF(x|l \ge 0) + \int_{Q}^{\infty}\Big(pQ\Big)dF(x|l \ge 0) - (\underline{w} - \tau(l))Q\Big]dG(\xi)\Bigg\}dH(l). \quad (7)
\end{aligned}$$

$$\begin{aligned}
&\int_{-\infty}^{0}\Big[\int_{\underline{\xi}}^{1/\rho}\Big(p - \underline{w} - (p - s)F(\xi\rho Q^*_{DR}|l \le 0)\Big)\rho\xi dG(\xi)\\
&\quad + \Big(p - \underline{w} - (p - s)F(Q^*_{DR}|l \le 0)\Big)\Big(1 - G(1/\rho)\Big)\Big]dH(l)\\
&\quad + \int_0^{\infty}\Big[\int_{\underline{\xi}}^{1/\rho}\Big(p - (\underline{w} - \tau(l)) - (p - s)F(\xi\rho Q^*_{DR}|l \ge 0)\Big)\rho\xi dG(\xi)\\
&\quad + \Big(p - (\underline{w} - \tau(l)) - (p - s)F(Q^*_{DR}|l \ge 0)\Big) \times \Big(1 - G(1/\rho)\Big)\Big]dH(l) = 0. \quad (8)
\end{aligned}$$

$$\begin{aligned}
\Pi_{DR}(Q|l \le 0) &= \int_{\underline{\xi}}^{1/\rho}\underbrace{\Big[\int_0^{\xi\rho Q}\Big(px + s(\xi\rho Q - x)\Big)dF(x|l \le 0) + \int_{\xi\rho Q}^{\infty}p\xi\rho Q dF(x|l \le 0)\Big]}_{=(a)}dG(\xi)\\
&\quad + \int_{1/\rho}^{\bar{\xi}}\underbrace{\Big[\int_0^{Q}\Big(px + s(\xi\rho Q - x)\Big)dF(x|l \le 0) + \int_{Q}^{\infty}\Big(pQ + s(\xi\rho Q - Q)\Big)dF(x|l \le 0)\Big]}_{=(a')}dG(\xi)\\
&\quad - (c - hl)\mu_\xi\rho Q, \quad (10)
\end{aligned}$$

$$\begin{aligned}
\Pi_{DR}(Q|l \ge 0) &= \int_{\underline{\xi}}^{1/\rho}\underbrace{\Big[\int_0^{\xi\rho Q}\Big(px + s(\xi\rho Q - x)\Big)dF(x|l \ge 0) + \int_{\xi\rho Q}^{\infty}p\xi\rho Q dF(x|l \ge 0)\Big]}_{=(b)}dG(\xi)\\
&\quad + \int_{1/\rho}^{\bar{\xi}}\underbrace{\Big[\int_0^{Q}\Big(px + s(\xi\rho Q - x)\Big)dF(x|l \ge 0) + \int_{Q}^{\infty}\Big(pQ + s(\xi\rho Q - Q)\Big)dF(x|l \ge 0)\Big]}_{=(b')}dG(\xi)\\
&\quad - c\mu_\xi\rho Q. \quad (11)
\end{aligned}$$

Proposition 4:

$$\Pi_{DR}(Q^*_{DR}) \leq \Pi_C(Z^*_C)$$

Proof: We observe that both (a) in Equation (10) and (b) in Equation (11) for $\xi \in [\underline{\xi}, 1/\rho]$ are consistent with the corresponding terms in Equation (1) (assuming that $Z^*_{DR}(Q) = \rho Q$). On the other hand, (a') in Equation (10) and (b') in Equation (11) for $\xi \in (1/\rho, \overline{\xi}]$ are not. Note that (a') or (b') can be rewritten as

$$\underbrace{\int_0^Q (p-s)xdF(x|l) + \int_Q^\infty (p-s)QdF(x|l) + s\xi\rho Q}_{=(c)} \tag{12}$$

and (c) above is nondecreasing in Q since $\frac{d(c)}{dQ} = \int_Q^\infty (p-s)dF(x|l) > 0$. For $\xi \in (1/\rho, \overline{\xi}]$ (which is the range of ξ corresponding to (a') or (b')), $Z^*_{DR}(Q^*_{DR}) = \rho Q^*_{DR} > \frac{Q^*_{DR}}{\xi}$ or $Q^*_{DR} < \xi Z^*_{DR}(Q^*_{DR})$. Thus, for $\xi > 1/\rho$, Equation (13) shown at the bottom of this page holds, which is the same as (a) or (b) when ρQ^*_{DR} is replaced by $Z^*_{DR}(Q^*_{DR})$.

In summary, $\Pi_{DR}(Q^*_{DR}) < \Pi_C(Z^*_{DR}(Q^*_{DR})) \leq \Pi_C(Z^*_C)$, where the first inequality is due to the discussion above, and the second inequality is due to the fact that Z^*_C is the maximizer of $\Pi_C(Z)$. □

The result above indicates that *double marginalization* still occurs in this decentralized supply chain under yield and lead-time uncertainty.

Lastly, we examine how the existence of yield uncertainty impacts the performance of this decentralized supply chain under the retailer-managed inventory strategy. We let $\Pi^R_{DR(PY)}(Q)$ be the expected profit function of the retailer under the assumption of a perfect yield. The function can then be expressed as Equation (14), shown at the bottom of this page.

It is straightforward to show that $\Pi^R_{DR(PY)}(Q)$ is concave in Q, hence the optimal order quantity $Q^*_{DR(PY)}$ under a perfect yield satisfies the following equation:

$$\int_{-\infty}^0 \Big(p - \underline{w} - (p-s)F(Q^*_{DR(PY)}|l \leq 0)\Big)dH(l) + \int_0^\infty \Big(p - (\underline{w} - \tau(l)) - (p-s)F(Q^*_{DR(PY)}|l \geq 0)\Big)dH(l) = 0. \tag{15}$$

The following proposition presents that the optimal order quantity under an uncertain yield Q^*_{DR} is larger than that under a perfect yield $Q^*_{DR(PY)}$, even when there exists lead-time uncertainty in the supply chain. We further examine how the variability of an uncertain lead-time would affect the size of the optimal order quantity under yield uncertainty in the numerical experiments.

Proposition 5: $Q^*_{DR(PY)} < Q^*_{DR}$.

Proof: See Appendix A-D. □

The proposition indicates that, if the retailer recognizes yield uncertainty during the production process, the retailer tends to increase the order quantity, which is larger than needed due to the fact that the larger order quantity influences the manufacturer's decision so that the retailer can reduce the risk of receiving less than what is needed. Due to the complicated expressions for the profit functions given above, we examine the effect of uncertain lead-time using numerical experiments.

IV. DECENTRALIZED SUPPLY CHAIN UNDER A VENDOR-MANAGED INVENTORY

In the previous section, we discussed a decentralized supply chain under the retailer-managed inventory strategy and evaluated the characteristics of expected profit functions and best responses for both the manufacturer and the retailer. We next investigate a decentralized supply chain under the vendor-managed inventory strategy where the manufacturer determines both the production and inventory decisions for the supply chain. In particular, we focus on a VMI-with-consignment contract model in which the manufacturer manages the retailer's inventory, but a payment is not made until a product is either used or sold. In this scenario, the sequence of the events is as follows:

1) Prior to the selling season, the manufacturer takes the market wholesale price $w(l)$ in the market for the retailer.
2) The manufacturer decides its production quantity Z. Due to the uncertain yield, the sellable quantity ξZ is less than or equal to Z. The sellable items ξZ are delivered to the retailer. Due to the uncertain production completion time, if production is completed before the

$$\begin{aligned}
&\int_0^{Q^*_{DR}} (p-s)xdF(x|l) + \int_{Q^*_{DR}}^\infty (p-s)Q^*_{DR}dF(x|l) + s\xi\rho Q^*_{DR} \\
&< \int_0^{\xi Z^*_{DR}(Q^*_{DR})} (p-s)xdF(x|l) + \int_{\xi Z^*_{DR}(Q^*_{DR})}^\infty (p-s)\xi Z^*_{DR}(Q^*_{DR})dF(x|l) + s\xi Z^*_{DR}(Q^*_{DR}) \\
&= \int_0^{\xi Z^*_{DR}(Q^*_{DR})} \Big(px + s(\xi Z^*_{DR}(Q^*_{DR}) - x)\Big)dF(x|l) + \int_{\xi Z^*_{DR}(Q^*_{DR})}^\infty p\xi Z^*_{DR}(Q^*_{DR})dF(x|l),
\end{aligned} \tag{13}$$

$$\begin{aligned}
\Pi^R_{DR(PY)}(Q) = &\int_{-\infty}^0 \left[\int_0^Q \Big(px + s(Q-x)\Big)dF(x|l \leq 0) + \int_Q^\infty pQdF(x|l \leq 0) - \underline{w}Q\right] dH(l) \\
&+ \int_0^\infty \left[\int_0^Q \Big(px + s(Q-x)\Big)dF(x|l \geq 0) + \int_Q^\infty pQdF(x|l \geq 0) - (\underline{w} - \tau(l))Q\right] dH(l).
\end{aligned} \tag{14}$$

promised delivery date, the manufacturer incurs holding costs per unit of time (h).

3) Customer demand X with retail price p is given to the retail channel, and the retailer makes a payment for the unit wholesale price $w(l)$ based on the actual, realized demand quantity.
4) At the end of the selling season, the manufacturer salvages unsold items at unit salvage value s.

Based on the sequence of events above, we first discuss the manufacturer's decision problem. For this, the expected profit function for the manufacturer in this scenario, $\Pi_{DV}^M(Z)$, is expressed as Equation (16), shown at the bottom of this page.

The following proposition presents the characteristics of the manufacturer's optimal production quantity given the exogenous wholesale price $w(l)$:

Proposition 6: Let $Z_{DV}^* = \arg\max_Z \Pi_{DV}^M(Z)$, the optimal production quantity of the manufacturer. The manufacturer's profit function $\Pi_{DV}^M(Z)$ is concave in Z, hence Z_{DV}^* holds the first-order optimality condition in Equation (17), shown at the bottom of this page.

The retailer has no decisions to make, and we simply state the retailer's expected profit $\Pi_{DV}^R(Z_{DV}^*)$, given the manufacturer's optimal production quantity Z_{DV}^*, as shown in Equation (18) at the bottom of this page.

We let $\Pi_{DV}(Z)$ be the total expected profit function for the decentralized supply chain under the VMI strategy, which is $\Pi_{DV}(Z) = \Pi_{DV}^M(Z) + \Pi_{DV}^R(Z)$. It is thus, straightforward to see that $\Pi_{DV}(Z) = \Pi_C(Z)$, hence the relationship $\Pi_{DV}(Z_{DV}^*) = \Pi_C(Z_{DV}^*) \leq \Pi_C(Z_C^*)$ holds. This also indicates that *double marginalization* occurs in this scenario under yield and lead-time uncertainty.

We now compare the production quantity under the VMI strategy with that from the centralized supply chain. Due to the introduction of the delay penalty $\tau(l)$ to the model, it is difficult to identify the relationship between the production quantities of two different systems in an analytical manner. In this section, we discuss the case when $\tau(l) = 0$ for

$$
\begin{aligned}
\Pi_{DV}^M(Z) &= \int_{-\infty}^{0} \Pi_{DV}^M(Z|l \leq 0)dH(l) + \int_{0}^{\infty} \Pi_{DV}^M(Z|l \geq 0)dH(l) \\
\text{where} \quad \Pi_{DV}^M(Z|l \leq 0) &= \int_{\underline{\xi}}^{\overline{\xi}} \left[\int_0^{\xi Z} \Big(\underline{w}x + s(\xi Z - x)\Big) dF(x|l \leq 0) \right. \\
&\quad \left. + \int_{\xi Z}^{\infty} \Big(\underline{w}\xi Z\Big) dF(x|l \leq 0) \right] dG(\xi) - (c - hl)\mu_\xi Z \\
\text{and} \quad \Pi_{DV}^M(Z|l \geq 0) &= \int_{\underline{\xi}}^{\overline{\xi}} \left[\int_0^{\xi Z} \Big((\underline{w} - \tau(l))x + s(\xi Z - x)\Big) dF(x|l \geq 0) \right. \\
&\quad \left. + \int_{\xi Z}^{\infty} \Big((\underline{w} - \tau(l))\xi Z\Big) dF(x|l \geq 0) \right] dG(\xi) - c\mu_\xi Z. \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
&\int_{-\infty}^{0} (\underline{w} - s) \left[\int_{\underline{\xi}}^{\overline{\xi}} \xi F(\xi Z_{DV}^* | l \leq 0) dG(\xi) \right] dH(l) + \int_0^{\infty} (\underline{w} - \tau(l) - s) \left[\int_{\underline{\xi}}^{\overline{\xi}} \xi F(\xi Z_{DV}^* | l \geq 0) dG(\xi) \right] dH(l) \\
&= \mu_\xi \left[(\underline{w} - c) + \int_{-\infty}^{0} hl dH(l) - \int_0^{\infty} \tau(l) dH(l) \right]. \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{DV}^R(Z_{DV}^*) &= \int_{-\infty}^{0} \Pi_{DV}^R(Z_{DV}^*|l \leq 0)dH(l) + \int_{0}^{\infty} \Pi_{DV}^R(Z_{DV}^*|l \geq 0)dH(l) \\
\text{where} \quad \Pi_{DV}^R(Z_{DV}^*|l \leq 0) &= \int_{\underline{\xi}}^{\overline{\xi}} \left[\int_0^{\xi Z_{DV}^*} (p - \underline{w}) x dF(x|l \leq 0) \right. \\
&\quad \left. + \int_{\xi Z_{DV}^*}^{\infty} (p - \underline{w}) \xi Z_{DV}^* dF(x|l \leq 0) \right] dG(\xi) \\
\text{and} \quad \Pi_{DV}^R(Z_{DV}^*|l \geq 0) &= \int_{\underline{\xi}}^{\overline{\xi}} \left[\int_0^{\xi Z_{DV}^*} \Big(p - (\underline{w} - \tau(l))\Big) x dF(x|l \geq 0) \right. \\
&\quad \left. + \int_{\xi Z_{DV}^*}^{\infty} \Big(p - (\underline{w} - \tau(l))\Big) \xi Z_{DV}^* dF(x|l \geq 0) \right] dG(\xi). \qquad (18)
\end{aligned}
$$

$l \geq 0$ (i.e., there is no penalty associated with a delay in production), which is the most advantageous situation for the manufacturer. In this case, Equation (17) becomes

$$\int_{-\infty}^{0}\left[\int_{\underline{\xi}}^{\overline{\xi}} \xi F(\xi Z_{DV}^{*}|l \leq 0)dG(\xi)\right]dH(l) + \int_{0}^{\infty}\left[\int_{\underline{\xi}}^{\overline{\xi}} \xi F(\xi Z_{DV}^{*}|l \geq 0)dG(\xi)\right]dH(l) = \left[\frac{(\underline{w}-c)+h\int_{-\infty}^{0} ldH(l)}{\underline{w}-s}\right]\mu_{\xi}. \quad (19)$$

We observe that the LHS of Equation (2) and Equation (19) are equivalent and, nondecreasing in Z. In addition,

$$\begin{aligned} &\text{RHS of Equation (2)} - \text{RHS of Equation (19)} \\ &= \left[\frac{(p-c)+h\int_{-\infty}^{0} ldH(l)}{p-s}\right]\mu_{\xi} - \left[\frac{(\underline{w}-c)+h\int_{-\infty}^{0} ldH(l)}{\underline{w}-s}\right]\mu_{\xi} \\ &= (p-\underline{w})\left[\frac{(c-s)-h\int_{-\infty}^{0} ldH(l)}{(p-s)(\underline{w}-s)}\right]\mu_{\xi} > 0, \end{aligned}$$

which immediately indicates that the production quantity in the decentralized supply chain under the VMI strategy is less than that in the centralized supply chain. The following proposition summarizes the discussion above.

Proposition 7: (i) *The expected total profit in a decentralized supply chain under VMI is less than that under a centralized supply chain (i.e., $\Pi_{DV}(Z_{DV}^{*}) \leq \Pi_{C}(Z_{C}^{*})$).*

(ii) *Suppose that $\tau(l) = 0$ for $l \geq 0$ (i.e., the unit wholesale price remains constant regardless of the delay). The production quantity in the decentralized supply chain under VMI is thus smaller than that in the centralized supply chain (i.e., $Z_{DV}^{*} < Z_{C}^{*}$).*

V. NUMERICAL EXPERIMENTS AND MANAGERIAL INSIGHTS

In this section, we conduct numerical experiments under the conditions listed in Table 3, and then we examine any changes in behavior in terms of order quantity, production quantity, and profit. In addition, we verify the aforementioned propositions, observe notable cases, and discuss appropriate recourse actions.

Table 3 presents the parameters and their values investigated in this paper. Note that the bracketed values are the default values in our experiments. For some parameters, there are specific constraints that are determined by the context of the experiment. For example, because the salvage value cannot exceed the unit cost, we consider it to be a ratio of the unit cost. In addition, the wholesale price is greater than or equal to the unit cost, and it cannot exceed 100, which is the retail price. Finally, we calculate the yield parameter using the following equations:

$$\underline{\xi} = 0.75 - \xi_G$$
$$\overline{\xi} = 0.75 + \xi_G.$$

A. EFFECTS OF PARAMETER CHANGES ON TOTAL PROFIT

We begin with the effects of parameter changes of the total profit of the supply chains discussed earlier. Table 4 shows the changes in total profit under the RMI and VMI strategies for different parameter values. As shown in Table 3, we vary the unit cost, holding cost, salvage ratio, lead-time factor, yield gap, and wholesale price to obtain the centralized supply chain total profit and the decentralized supply chain total profit for the RMI and VMI models.

As proven in Propositions 4 and 7, we observe that the total profit for centralized supply chain is always greater than or equal to that for the decentralized supply chain. In addition, total profit for a VMI is generally higher than that for a RMI. For both the total profit for the centralized supply chain and the total profit for the decentralized supply chain under RMI and VMI, we can see that total profit increases as the unit cost decreases, the holding cost decreases, the salvage ratio increases, the lead-time factor decreases, and the yield gap decreases. Particularly notable is that, as lead-time and yield uncertainty decreases, the total profit increases by a significant margin for all the supply chains. However, for a change in the wholesale price, the RMI and VMI strategies exhibit opposing trends. For the VMI strategies, total profit increases slightly as wholesale price rises. In contrast, the total profit under RMI decreases rapidly in the same circumstances. We discuss this trend further in Section V-E.

B. EFFECTS OF YIELD ON ORDER QUANTITY Q

In this section, we compare order quantity Q for different yield levels while varying the unit cost, holding cost, wholesale price, and the lead-time factor. Detailed results of the experiment are shown in Table 5. As proven in Proposition 5, we see that for all cases, the derived order quantity is higher than that under the perfect yield scenario.

We believe that yield has a significant influence on production decision-making. The retailer can utilize information on the manufacture's yield to determine the optimal quantity Q. If yield uncertainty is high, the retailer, aware of the fact that they might not receive the ordered goods, will order more than necessary. Conversely, if yield uncertainty is low, the retailer

TABLE 3. Parameters for the numerical experiments.

	Values	Description
c	5, 15, [25], 35, 45	unit production cost
p	100	unit retail price
h	0.00001, 0.00005, [0.0001], 0.0005, 0.001	unit holding cost per unit
s	$0.1c$, $0.3c$, [$0.5c$], $0.7c$, $0.9c$	unit salvage value
b	1000	max lead-time
$\underline{w}$	55, 65, [75], 85, 95	unit wholesale price
ϕ	0.051, 0.10, [0.15], 0.20, 0.25	variability factor for lead-time
ξ_G	0.051, 0.10, [0.15], 0.20, 0.25	yield gap

TABLE 4. Experiment results for total profit.

Parameter	Value	Centralized	Decentralized RMI	Decentralized VMI
Unit cost	5	44204.53	20740.59	44192.51
	15	37072.48	18974.07	37013.45
	25	30457.34	17122.43	30304.00
	35	24254.30	15149.60	23893.40
	45	18485.32	13009.57	17717.32
Holding cost	0.00001	30460.10	17123.44	30306.82
	0.00005	30458.87	17122.99	30305.57
	0.0001	30457.34	17122.43	30304.00
	0.0005	30445.10	17117.96	30291.42
	0.001	30429.79	17112.37	30275.71
Salvage value	2.5	27323.57	15251.00	27007.76
	7.5	28801.81	16123.27	28568.74
	12.5	30457.34	17122.43	30304.00
	17.5	32344.97	18290.07	32262.92
	22.5	34619.34	19698.12	34595.53
Lead-time factor value	0.05	31314.76	17304.29	31170.62
	0.1	30893.81	17221.38	30745.41
	0.15	30457.34	17122.43	30304.00
	0.2	30005.70	17006.11	29847.43
	0.25	29539.44	16870.84	29376.38
Yield gap	0.05	30806.61	17446.48	30659.81
	0.1	30672.80	17287.39	30524.39
	0.15	30457.34	17122.43	30304.00
	0.2	30167.34	16951.20	30006.83
	0.25	29814.12	16772.62	29639.63
Wholesale price	55	30457.34	25479.11	29451.23
	65	30457.34	21806.17	30049.07
	75	30457.34	17122.43	30304.00
	85	30457.34	11411.38	30413.50
	95	30457.34	4661.30	30452.70

will only order the necessary amount. During this decision process, yield uncertainty is thus crucial.

In addition, according to Proposition 2, the response factor is dependent on yield; as yield increases, we see that the response factor approaches 1 (i.e, the order quantity equals optimal production quantity). The response factor represents the ratio between Q (order quantity) and Z (optimal production quantity) considering yield. Lower yields lead the retailer to order higher quantities, which eventually leads the manufacturer to produce on a higher production quantity. In other words, a lower yield leads to additional bullwhip effects. In fact, under perfect yield, the response factor is equal to 1 and the retailer will receive the same amount as they ordered.

C. EFFECTS OF VMI REPLENISHMENT ON PRODUCTION QUANTITY Z

In this section, we compare production quantity between the centralized supply chain model and the decentralized supply chain model under VMI. Table 6 shows the experimental results for production quantity Z. Similar to the previous experiments, we vary the unit cost, holding cost, salvage ratio, lead-time factor, yield gap, and wholesale price. The results of the experiment show that for all cases, the centralized production quantity surpasses the production quantity under the VMI model.

We see that production quantity increases as the unit cost, holding cost, lead-time factor, and yield gap decreases. In contrast, for salvage value and wholesale price, we observe that production quantity increases as both factors decrease. As unit cost decreases and wholesale price increases, the manufacturer's profit increases, and they will tend to increase production. The lead-time and yield gap are elements of uncertainty. We observe that a shorter lead-time will encourage the manufacturer to increase production as goods are more likely to be sold in the current season. If the yield gap is lower, then the manufacturer will also try to increase production because overproduction expenses and the opportunity cost from underproduction decrease. In addition, as salvage value increases, the manufacturer will also increase production because the loss from unsold goods decreases.

D. EFFECTS OF UNIT COST VARIATION

In this experiment, we only vary the unit cost while keeping all other variables constant. Note that the manufacturer's profit is determined as either the difference between sales and costs (centralized) or the difference between wholesale price and costs (decentralized). We assert that the high production quantity is due to the fact that, as profit increases, the retailer is able to deal with more leftover products arising from uncertainty in demand and lead-time. As shown in Figure 2, as the unit cost decreases, total profit increases under the decentralized supply chain model. Because most of the increased profit is then distributed to the manufacturer, the manufacturer now has an incentive to aggressively increase

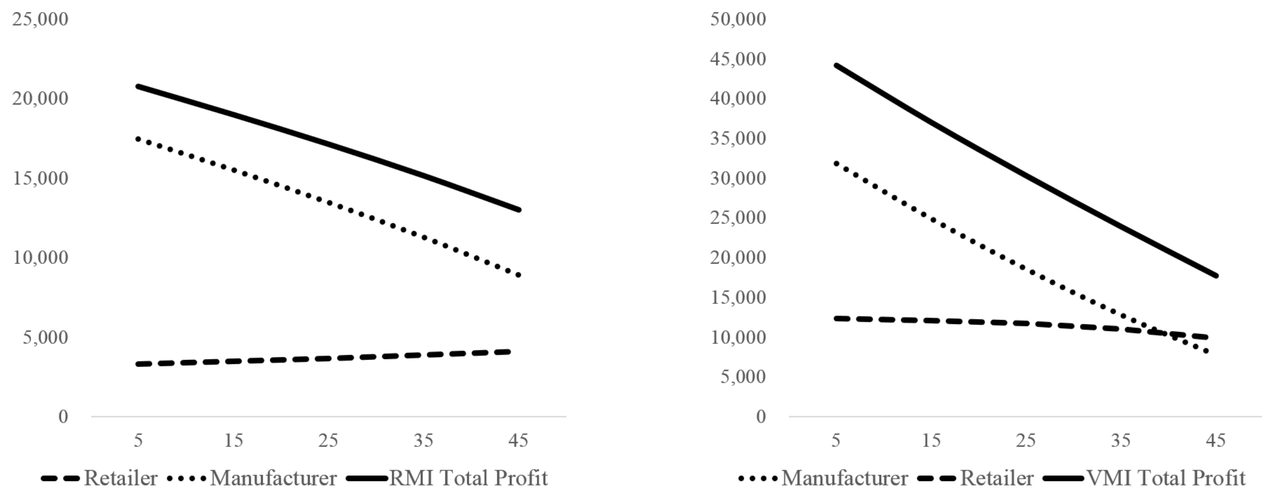
TABLE 5. Experiment results for order quantity Q.

Parameter	Value	Order quantity Q with perfect yield	Order quantity Q with Yield gap				
			±0.1	±0.2	±0.3	±0.4	±0.5
Unit cost	5	254.45	254.47	254.51	254.56	254.64	254.75
	15	268.20	268.47	268.83	269.32	269.98	270.84
	25	283.53	284.41	285.55	286.97	288.72	290.82
	35	300.71	302.82	305.36	308.32	311.70	315.43
	45	320.11	324.44	329.30	334.60	340.25	346.10
Holding cost	0.00001	283.53	284.41	285.55	286.97	288.72	290.82
	0.00005	283.53	284.41	285.55	286.97	288.72	290.82
	0.0001	283.53	284.41	285.55	286.97	288.72	290.82
	0.0005	283.53	284.42	285.55	286.98	288.73	290.83
	0.001	283.53	284.42	285.56	286.99	288.75	290.85
Salvage value	2.5	254.45	256.29	258.50	261.07	263.99	267.21
	7.5	268.20	269.58	271.28	273.33	275.74	278.48
	12.5	283.53	284.41	285.55	286.97	288.72	290.82
	17.5	300.71	301.12	301.67	302.41	303.37	304.60
	22.5	320.11	320.17	320.26	320.38	320.56	320.82
Lead-time factor	0.05	285.24	286.12	287.24	288.65	290.39	292.47
	0.1	284.52	285.40	286.53	287.95	289.69	291.78
	0.15	283.53	284.41	285.55	286.97	288.72	290.82
	0.2	282.24	283.13	284.27	285.70	287.46	289.55
	0.25	280.64	281.53	282.67	284.11	285.86	287.96
Wholesale price	55	500.47	503.77	507.75	512.42	517.78	523.75
	65	392.00	393.72	395.87	398.48	401.60	405.22
	75	283.53	284.41	285.55	286.97	288.72	290.82
	85	175.05	175.46	176.00	176.69	177.55	178.61
	95	66.58	66.70	66.86	67.07	67.34	67.68

TABLE 6. Experiment results for production quantity Z.

Parameter	Value	production quantity Z under Centralized	production quantity Z under VMI
Unit cost	5	1358.29	1320.59
	15	1192.88	1135.58
	25	1088.78	1008.46
	35	995.41	873.74
	45	896.32	713.62
Holding cost	0.00001	1088.84	1008.53
	0.00005	1088.81	1008.50
	0.0001	1088.78	1008.46
	0.0005	1088.54	1008.14
	0.001	1088.23	1007.75
Salvage value	2.5	971.60	867.11
	7.5	1025.26	932.20
	12.5	1088.78	1008.46
	17.5	1173.13	1105.00
	22.5	1329.90	1275.47
Lead-time factor	0.05	1114.6	1038.09
	0.1	1101.47	1023.23
	0.15	1088.78	1008.46
	0.2	1076.62	994.10
	0.25	1065.11	980.31
Yield gap	0.05	1095.55	1019.80
	0.1	1092.57	1015.30
	0.15	1088.78	1008.46
	0.2	1085.28	1000.04
	0.25	1084.98	990.24
Wholesale price	55	1088.78	886.88
	65	1088.78	958.94
	75	1088.78	1008.46
	85	1088.78	1045.47
	95	1088.78	1074.59

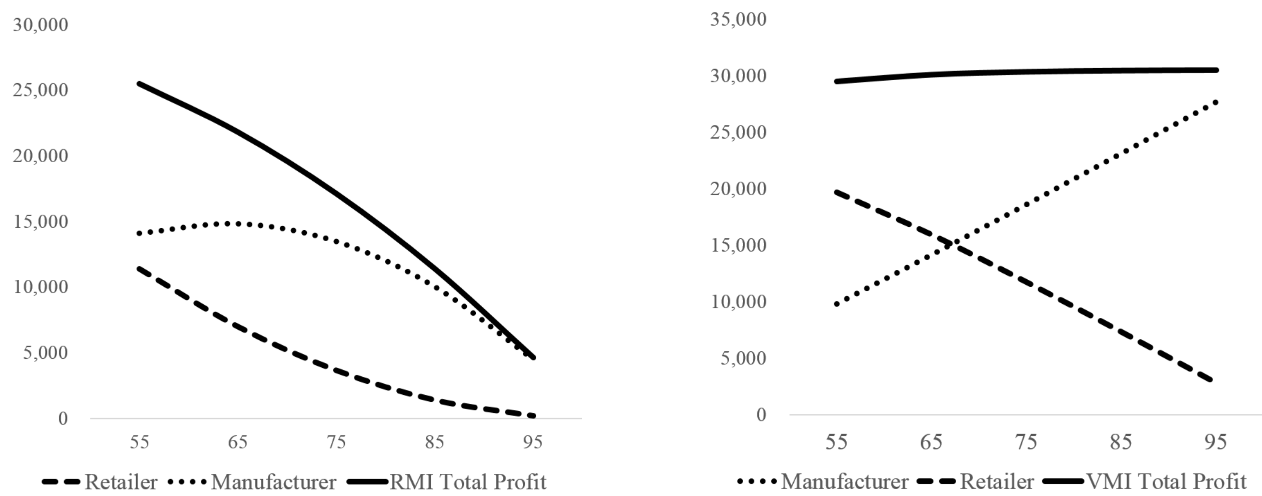
production quantity. Under both the RMI and VMI models, as the unit cost increases, total profit and the manufacturer's profit decrease. For the retailer, however, we observe quite different results. In the VMI model, the retailer can only sell the quantity produced by the manufacturer. Therefore, we observe a decrease in the retailer's profit. In contrast, in the RMI model, we notice that the retailer's profit increases as the unit cost increases. We assert that this trend is due to the fact that the salvage value is calculated as a percentage of the unit cost. As a result, as the unit cost goes up, the salvage



(a) Decentralized RMI

(b) Decentralized VMI

FIGURE 2. Profit comparison in terms of unit cost variation.



(a) Decentralized RMI

(b) Decentralized VMI

FIGURE 3. Profit comparison in terms of whole sale price variation.

value increases accordingly. The retailer can now sell unsold goods at a higher price, which in return mitigates the demand uncertainty risk. Eventually, as we have seen in Section V-B, the retailer will increase the order quantity and in result, increase the retailer's profit as well. Although one might want to decrease the unit cost in order to increase the total RMI profit, this may not be desirable for the retailer as their profit will decrease. Therefore, in order to overcome this conflict of interest, we suggest adopting either an appropriate contract or the guarantee of a minimum salvage value.

E. EFFECTS OF WHOLESALE PRICE VARIATION

For this experiment, we only vary the wholesale price while keeping the other parameters fixed. Because the wholesale price determines the percentage of the profit $(p - c)$ that the manufacturer and retailer share, we can easily see that the wholesale price is an important indicator of the profit distribution between the manufacturer and the retailer. As shown in Figure 3, we observe that, as the wholesale price increases, the retailer's profit decreases. This is trivial, however, because the retailer's profit is calculated by subtracting the wholesale price from the price. Hence, as the wholesale price increases, the retailer's profit decreases and eventually the order quantity Q falls as well. For the RMI model, as the wholesale price increases, the retailer's order quantity decreases in return. As a result, the manufacturer's production quantity also decreases, which eventually causes the manufacturer's profit to decrease as well. In contrast, for the VMI model, we observe that, because the manufacturer's profit $(\underline{w} - c)$ increases as the wholesale price increases, the manufacturer would want to increase production quantity as much as possible. However, from the retailer's perspective, due to the fact that the retailer's profit is determined by subtracting the wholesale price from the retail price, the retailer's profit will decrease rapidly despite an increase in sales. Note that, for the VMI model, maintaining a high wholesale price is generally desirable as it will lead to higher levels of total profit. However, this cannot always be done because it will lead to a dramatic decrease in the retailer's profit. Therefore, introducing an appropriate contract is desirable as it will not only maximize the total supply chain profit, but also resolve the aforementioned conflict between the retailer and the manufacturer.

VI. CONCLUSION

In this study, we examine models that analyze the impact of supply uncertainty in retailer-managed and vendor-managed decentralized supply chains on supply chain performance. We thus identify optimal production and order quantities for centralized and decentralized supply chains under lead-time, yield, and demand uncertainty. For the RMI model, when yield is low, the retailer always attempts to order more than the optimal quantity, which might lead to the bullwhip effect. Therefore, in order to minimize the impact of the bullwhip effect, the manufacturer wishes to obtain and utilize its yield information so that they can better gauge the exact level of demand.

The centralized production quantity is always greater than the production quantity of the VMI, which is quite intuitive. Under the same conditions, it would be more advantageous for the manufacturer under the VMI model to set the wholesale price higher; however, because the retailer's profit decreases rapidly as the wholesale price increases, there needs to be an appropriate agreement to ensure a fair distribution. As we have confirmed in this study, under the decentralized supply chain model, the total profit of the VMI and RMI models increases as the unit cost decreases, the holding cost decreases, the salvage value increases, and as the lead-time and yield uncertainty decrease. Therefore, we confirm through the numerical studies that higher yield or lead-time uncertainty generally leads to lower expected profits for both the manufacturer and retailer no matter which inventory management policy is utilized. Overall, the profit for the RMI model is higher than that for the VMI model. In addition, for certain parameters (unit cost and wholesale price), we observe that, although total profit increases or remains the same, there is a conflict of interest between the retailer and the manufacturer.

One of the limitations of this study is that we do not propose appropriate contracts that coordinate a decentralized supply chain under either the RMI or VMI model when both yield and lead-time uncertainty exists. Therefore, we believe that the result of our study can be used as the foundation for in-depth research into supply chain contracts. Another limitation is that we assume that the retailer knows the manufacturer's yield and lead-time information. Thus, our study can be extended to a decentralized supply chain where this information is not fully available to the retailer. In addition, we consider a supply chain consisting of a single manufacturer and a single retailer. Thus, our study can be extended to more complex supply chains (e.g., multiple retailers or three-echelon supply chains including a distributor) and evaluate the impact of simultaneous yield and lead-time uncertainty.

APPENDIX A
PROOFS OF PROPOSITIONS

A. PROOF OF PROPOSITION 1

Proof: By the Lebniz's Rule, the first-order and second-order derivatives of $\Pi_C(Z)$ with respect to Z are given in Equation (20), shown at the bottom of this page, respectively. Thus, the results above indicate that the function $\Pi_C(Z)$ is concave in Z, and hence Z_C^* satisfies the first-order condition. □

B. PROOF OF PROPOSITION 2

Proof: Using the Leibniz Rule, the first-order and second-order derivatives of $\Pi_{DR}^M(Z;Q)$ with respect to Z are given in Equation (21), shown at the bottom of the next page, respectively. We remark that the second-order derivative is negative due to the assumption $\underline{w} - \int_0^\infty \tau(l)dH(l) > s$. Therefore, the function $\Pi_{DR}^M(Z;Q)$ is concave in Z, hence the best response Z_{DR}^* should satisfy the first-order optimality condition $\left.\frac{d\Pi_{DR}^M(Z;Q)}{dZ}\right|_{Z=Z_{DR}^*} = 0.$ □

C. PROOF OF PROPOSITION 3

Proof: Taking the first-order and second-order derivatives on $\Pi_{DR}^R(Q)$ with respect to Q yields Equation (22), shown at the bottom of the next page, respectively, implying that $\Pi_{DR}^R(Q)$ is concave in Q. We note that ρ is independent

$$\frac{d\Pi_C(Z)}{dZ} = \left((p-c) + h\int_{-\infty}^{0} l dH(l)\right)\mu_\xi - (p-s)\left[\int_{-\infty}^{0}\left(\int_{\underline{\xi}}^{\overline{\xi}} \xi F(\xi Z|l \le 0)dG(\xi)\right)dH(l) + \int_0^\infty \left(\xi F(\xi Z|l \ge 0)dG(\xi)\right)dH(l)\right]$$

$$\text{and}\quad \frac{d^2\Pi_C(Z)}{dZ^2} = -(p-s)\left[\int_{-\infty}^{0}\left(\int_{\underline{\xi}}^{\overline{\xi}} \xi^2 f(\xi Z|l \le 0)dG(\xi)\right)dH(l) + \int_0^\infty \left(\xi^2 f(\xi Z|l \ge 0)dG(\xi)\right)dH(l)\right] < 0 \quad (20)$$

of Q. Thus, the optimal order quantity Q^*_{DR} satisfies the first-order optimality condition $\frac{d\Pi^R_{DR}(Q)}{dQ}\Big|_{Q=Q^*_{DR}} = 0.$ □

D. PROOF OF PROPOSITION 5

Proof: Observe Equation (23), shown at the bottom of this page, where the inequality is due to the fact that c.d.f. $F(\cdot)$ is a nondecreasing function, and the equality is due to Equation (15). Moreover, Equation (24) shown at the bottom of this page holds.

Both results above immediately indicate that

$$\frac{d\Pi^R_{DR}(Q)}{dQ}\Bigg|_{Q=Q^*_{DR(PY)}} > 0$$

$$\text{while} \quad \frac{d\Pi^R_{DR}(Q)}{dQ}\Bigg|_{Q=Q^*_{DR}} = 0.$$

Therefore, the concavity of $\Pi^R_{DR}(Q)$ in Q implies $Q^*_{DR(PY)} < Q^*_{DR}$. □

$$\begin{aligned}
\frac{d\Pi^M_{DR}(Z;Q)}{dZ} &= \int_{-\infty}^{0}\left[\int_{\underline{\xi}}^{Q/Z}(\underline{w}-c+hl)\xi dG(\xi) - \int_{Q/Z}^{\overline{\xi}}(c-hl-s)\,\xi dG(\xi)\right]dH(l)\\
&\quad+\int_{0}^{\infty}\left[\int_{\underline{\xi}}^{Q/Z}\big(\underline{w}-\tau(l)-c\big)\,\xi dG(\xi) - \int_{Q/Z}^{\overline{\xi}}(c-s)\,\xi dG(\xi)\right]dH(l)\\
\text{and}\quad \frac{d^2\Pi^M_{DR}(Z;Q)}{dZ^2} &= -\int_{-\infty}^{0}\left[(\underline{w}-s)\frac{Q^2}{Z^3}g\left(\frac{Q}{Z}\right)\right]dH(l) - \int_{0}^{\infty}\left[(\underline{w}-\tau(l)-s)\frac{Q^2}{Z^3}g\left(\frac{Q}{Z}\right)\right]dH(l) < 0
\end{aligned}\tag{21}$$

$$\begin{aligned}
\frac{d\Pi^R_{DR}(Q)}{dQ} &= \int_{-\infty}^{0}\Big[\int_{\underline{\xi}}^{1/\rho}\Big(p-\underline{w}-(p-s)F(\xi\rho Q|l\le 0)\Big)\rho\xi dG(\xi)\\
&\quad+\Big(p-\underline{w}-(p-s)F(Q|l\le 0)\Big)\Big(1-G(1/\rho)\Big)\Big]dH(l)\\
&\quad+\int_{0}^{\infty}\Big[\int_{\underline{\xi}}^{1/\rho}\Big(p-(\underline{w}-\tau(l))-(p-s)F(\xi\rho Q|l\ge 0)\Big)\rho\xi dG(\xi)\\
&\quad+\Big(p-(\underline{w}-\tau(l))-(p-s)F(Q|l\ge 0)\Big)\Big(1-G(1/\rho)\Big)\Big]dH(l)\\
\frac{d^2\Pi^R_{DR}(Q)}{dQ^2} &= -\int_{-\infty}^{0}\Big[\int_{\underline{\xi}}^{1/\rho}\Big((p-s)\rho^2\xi^2 f(\xi\rho Q|l\le 0)\Big)dG(\xi)+(p-s)f(Q|l\le 0)\Big(1-G(1/\rho)\Big)\Big]dH(l)\\
&\quad-\int_{0}^{\infty}\Big[\int_{\underline{\xi}}^{1/\rho}\Big((p-s)\rho^2\xi^2 f(\xi\rho Q|l\ge 0)\Big)dG(\xi)+(p-s)f(Q|l\ge 0)\Big(1-G(1/\rho)\Big)\Big]dH(l) < 0
\end{aligned}\tag{22}$$

$$\begin{aligned}
&\int_{-\infty}^{0}\left[\int_{\underline{\xi}}^{1/\rho}\Big(p-\underline{w}-(p-s)F(\xi\rho Q^*_{DR(PY)}|l\le 0)\Big)\rho\xi dG(\xi)\right]dH(l)\\
&\quad+\int_{0}^{\infty}\left[\int_{\underline{\xi}}^{1/\rho}\Big(p-(\underline{w}-\tau(l))-(p-s)F(\xi\rho Q^*_{DR(PY)}|l\ge 0)\Big)\rho\xi dG(\xi)\right]dH(l)\\
&> \int_{-\infty}^{0}\left[\int_{\underline{\xi}}^{1/\rho}\Big(p-\underline{w}-(p-s)F(Q^*_{DR(PY)}|l\le 0)\Big)\rho\xi dG(\xi)\right]dH(l)\\
&\quad+\int_{0}^{\infty}\left[\int_{\underline{\xi}}^{1/\rho}\Big(p-(\underline{w}-\tau(l))-(p-s)F(Q^*_{DR(PY)}|l\ge 0)\Big)\rho\xi dG(\xi)\right]dH(l)\\
&= \left(\int_{\underline{\xi}}^{1/\rho}\rho\xi dG(\xi)\right)\Big[\int_{-\infty}^{0}\Big(p-\underline{w}-(p-s)F(Q^*_{DR(PY)}|l\le 0)\Big)dH(l)\\
&\quad+\int_{0}^{\infty}\Big(p-(\underline{w}-\tau(l))-(p-s)F(Q^*_{DR(PY)}|l\ge 0)\Big)dH(l)\Big]\\
&= \;0
\end{aligned}\tag{23}$$

$$\begin{aligned}
&\Big(1-G(1/\rho)\Big)\Big[\int_{-\infty}^{0}\Big(p-\underline{w}-(p-s)F(Q^*_{DR(PY)}|l\le 0)\Big)dH(l)\\
&\quad+\int_{0}^{\infty}\Big(p-(\underline{w}-\tau(l))-(p-s)F(Q^*_{DR(PY)}|l\ge 0)\Big)dH(l)\Big] = 0.
\end{aligned}\tag{24}$$

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